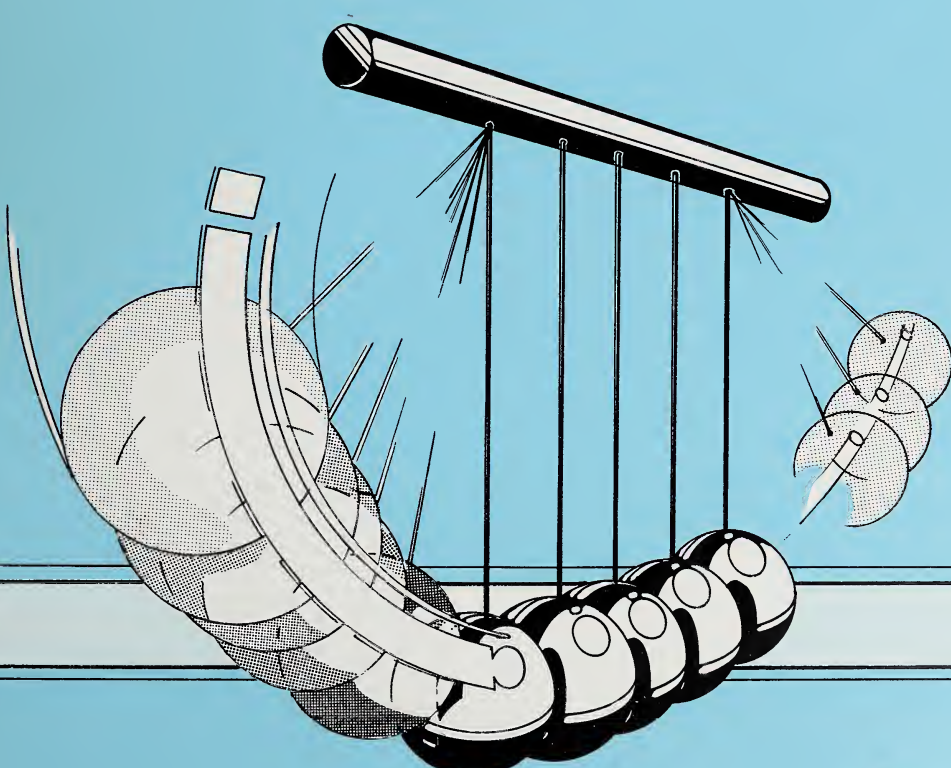
PHYSICS UPGRADING



Alberta
EDUCATION

Physics Upgrading

Lessons A-J

Distance Learning

Alberta
EDUCATION

Physics Upgrading
Student Module
Lessons A-J and Answers
Alberta Correspondence School
ISBN No. 0-7741-0065-6

INTRODUCTION

Purpose and Outline of the Course

The Physics 30A course has been designed to provide the adult student with a grounding in the basic concepts of physics. Many adult students who have been away from the classroom for several years do not have a sufficient background in physics to enable them to handle Physics 30.

The material in this course consists of ten lessons, designated A - J. This course deals with basic concepts usually covered in Physics 10 and Physics 20. It is hoped that these lessons will help students to prepare themselves adequately for the Physics 30 course. Students will not be tested on the material covered in Lessons A to J.

Reference Material

All materials in Lessons A to J are self-contained, so no textbook is required. However, if you would like to do extra reading, you could consult one of the following books:

1. **Fundamentals of Physics:** Heath, Macnaughton and Martindale, D.C. Heath Canada Ltd, Toronto
2. **Physics: An Experimental Science;** White, White and Gould, D Van Nostrand Company, Inc. Toronto
3. **Physics: A Human Endeavour. Units 1 and 3.** Paul, Peirce, and Stief, Holt, Rinehart and Winston of Canada, Limited. Toronto.

General Instructions

1. As the first ten lessons do not constitute a credit course and as there will be no test to promote the student into the Physics 30 course, you will be expected to rely mainly upon your own endeavor and self-direction to obtain the maximum benefit from the course.
2. **Problems** constitute the major part of physics. Orderly procedure is absolutely **necessary** when solving problems. The following steps are recommended.
   (a) Master the typical illustrative example given in the lesson.
   (b) Read the assigned problem through at least twice, **noting** the **information given** and the information which is required.

(c) The problem should be read carefully and studied so that the problem situation may be visualized, then clearly expressed, if possible, in a drawn diagram. All known factors should be indicated on the diagram and all quantities to be evaluated (unknowns) should be recognized and indicated as unknowns on or near the diagram.

(d) Decide upon which relationship between the known and the required information must be used in order to solve the problem. If this relationship can be expressed in **formula form**, write down this formula.

(e) Change the **subject** of the formula as required. This means solving the formula for the quantity sought: by transposition, cross-multiplication, etc. For example if you want to find the acceleration of a body from the formula $\mathbf{d = \frac{1}{2}at^2}$, you will solve the formula for $\mathbf{a}$: $\mathbf{\frac{1}{2}at^2 = d}$; whence $\mathbf{a = \frac{d}{\frac{1}{2}(t^2)}}$. This is a more efficient method than trying to solve for the unknown in the original formula.

3. Substitute the known values from the problem; make sure that all values are in **CONSISTENT** units.

   You can make your mathematical work easier by cancellation or by some other short cuts. Like units in the numerator will cancel like units in the denominator in many problems, leaving your answer with the correct units.

4. Solve the problem and express the answer in the appropriate units.

5. Ask yourself if the answer is reasonable in view of the information supplied in the problem, and if it contains the information demanded by the question.

ANSWERS to **Self-Check Exercises** are given in each lesson. You will gain most from the **Self-Check Exercises** if you do each problem or question as completely as possible before you refer to the answer.

Note: A metric ruler and a protractor are needed for this course.

Outline of the Phyics Upgrading Course (Lessons A-J)

The topics covered in each lesson are given below.

Lesson A

Scientific Notation
The Metric System
Trigonometry
Scientific Models

Lesson B

Equations of Motion
Motion and Direction

Lesson C

Vectors
Vector Diagrams

Lesson D

Force
Mass
Inertia

Lesson E

Newton's Second Law
Newton's Third Law
Newton's Law of Gravitation

Lesson F

Momentum
Work
Energy
Power

Lesson G

Kinetic Energy
Potential Energy
Conservation of Mechanical Energy

Lesson H

Kinetic Molecular Theory
Heat and Friction
Heat and Temperature
Conservation of Energy

Lesson I

Waves as Energy Carriers
Characteristics of Waves

Lesson J

Reflection and Refraction of Waves
Diffraction and Interference of Waves

We wish you success and enjoyment in this course.

EQUATIONS

Kinematics

$$\vec{v}_{ave} = \frac{\vec{d}}{t}$$

$$\vec{a} = \frac{\vec{v}_f + \vec{v}_i}{t}$$

$$\vec{d} = \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)t$$

$$v_f^2 = v_i^2 + 2ad$$

Dynamics

$$\vec{F} = m\vec{a}$$

$$\vec{F}t = m\Delta\vec{v}$$

$$\vec{F}_g = m\vec{g}$$

$$F_g = \frac{Gm_1m_2}{R^2}$$

Momentum & Energy

$$\vec{p} = m\vec{v}$$

$$W = Fd$$

$$P = \frac{W}{t}$$

$$E_k = \frac{1}{2}mv^2$$

$$E_p = mgh$$

Waves & Light

$$v = f\lambda$$

$$T = \frac{1}{f}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

Physics Upgrading

Booklet of Solutions and Answers

Dear Student:

You will not be required to send in lessons A-J in Physics Upgrading. Attached you will find a booklet of answers and solutions. Study these lessons very carefully, work on the exercises and then check your own work by using this booklet.

When you feel you have mastered these lessons you should proceed to doing the Physics 30 lessons. However if you have difficulty with some concepts you may call us by the government RITE number in your area, providing you are an Alberta resident.

We wish you success in this course.

Ronald Billey

Ronald Billey
Vice Principal
Math and Science Department

The answers below are for **LESSON A, page 2.**

1. (a) 2

   (b) 3
   3
   3

   (c) 4
   4
   4

2. (a) 2
   2

   (b) 3
   3

   (c) 4
   4

   (d) 5
   5

3. (a) 1.53×10^2
   (b) 2.86×10^2
   (c) 8.592×10^3
   (d) 1.0158×10^4
   (e) 2.78643×10^5
   (f) 8.953295×10^6

LESSON A, page 4

1. **(b)** 2
   −2 −2

   (c) 2
   −2 −2

   (d) 3
   −3 −3

   (e) 4
   −4 −4

LESSON A, page 5

1	-1
2	-2
3	-3
4	-4
5	-5

LESSON A, page 7

1. No, the bases are different.

2. (a) 2^7 (d) 10^4
   (b) 3^{10} (e) 10^{12}
   (c) 7^{13} (f) $10^{3/2}$ $(10^{½+1})$

3. (a) 2^3 (d) 10^{-6}
   (b) 3^6 (e) 10^6
   (c) 7^{-1} (f) $10^{-½}$ $(10^{½-1})$

   For (f), note that $10^{½}$ is 10 = 3.16, so $\dfrac{10^{½}}{10}$ is $\dfrac{3.16}{10}$ = 0.316; but $10^{-½}$ is $\dfrac{1}{10^{½}} = \dfrac{1}{3.16}$
   $= 0.316$

4. (a) 1×10^{-5} (d) 2×10^{-2}
   (b) 1×10^8 (e) 5×10^{11}
   (c) 1×2^{-3} (f) 3×10^{-80}

   Note that the numerators do not change.

LESSON A, page 8

1. (a) 2×10^7
   (b) $14 \times 10^{10} = 1.4 \times 10^{11}$
   (c) $10 \times 10^9 = 1.0 \times 10^{10}$
   (d) $81 \times 10^{18} = 8.1 \times 10^{19}$

   Note that the first number is 1 or greater but less than 10.

LESSON A, page 9

2. (a) $\frac{1}{5} \times 10^2 = 0.20 \times 10^2 = 2.0 \times 10^1 = \mathbf{20}$

(b) 3.0×10^{-6}

(c) 0.40

(d) $\frac{2.56}{1.6} = 1.6$, therefore $\frac{2.56 \times 10^{17}}{1.6 \times 10^{-17}} = 1.6 \times 10^{17-(-17)} = \mathbf{1.6 \times 10^{34}}$

3. $\frac{108 \times 10^4}{12.0 \times 10^2} = \mathbf{9.00 \times 10^2}$

For the rules regarding significant digits, see Lesson one, pages 17, 18, and 19 in the regular Physics 30 course.

LESSON A, page 17

2. $= 0.35 \times 1000$ g $\quad 0.35 \text{ kg} \times \frac{1000 \text{ g}}{\text{kg}}$
$= \mathbf{3.5 \times 10^2}$ **g**

3. $= 1500 \times 10^{-3}$ km $\quad 1500 \text{ m} \times \frac{10^{-3} \text{ km}}{\text{m}}$
= **1.5 km**

Note that the 'm' cancels out, and that 10^{-3} km = 1 m. You could also have multiplied by $\frac{1 \text{ km}}{1000 \text{ m}}$.

4. $= 100 \times 10^{-3}$ L $\quad 100 \text{ mL} \times \frac{1 \text{ L}}{1000 \text{ mL}}$
= **0.10 L**

5. $= 10^9 \times 10^{-6}$ MΩ $\quad 10^9\Omega \times \frac{1\text{M}\Omega}{10^6\Omega}$

$= \mathbf{10^3 M\Omega}$

(6 and 7 are given)

8. $= \mathbf{1.0 \times 10^6}$ **cm^3** $\quad (10^2)^3 = 10^{2 \times 3} = 10^6$

9. = **10 m/s** $\quad 36\frac{\text{km}}{\text{h}} \times \frac{1 \text{ h}}{3600\text{s}} \times \frac{1000\text{m}}{\text{km}}$

LESSON A, page 21

1. $\frac{h}{100\text{ m}} = \tan 38°$

   $\therefore$ h = 100 m tan 38°
   h = 100 m (0.7813)
   h = 78 m (to 2 significant digits)

2. 90° − 75° = 15°

   $\frac{x}{50\text{ m}}$ tan 15°

   $\therefore$ x = 50 m tan 15°
   x = 50 m (0.2679)
   x = 13 m

LESSON A, page 22

3. $\tan A = \frac{9\text{ m}}{15\text{ m}} = 0.60$

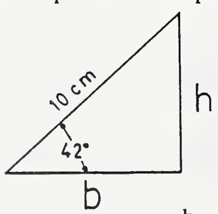
   $\therefore$ **A = 31°**

   (Note that the 'm' cancels out, so tan A is a unitless number)

   $\tan B = \frac{15\text{ m}}{9\text{ m}} = 1.67$

   $\therefore$ **B = 59°**

   Alternate method: B = 90° − 31° = 59°
   Other methods using sines and the hypotenuse could also be used, but they would require extra steps.

4. 

   $\sin 42° = \frac{h}{10.0\text{ cm}}$

   $\therefore$ h = 10.0 cm (0.6691)
   h = **6.69 cm**

   $\cos 42° = \frac{b}{10.0\text{ cm}}$

   $\therefore$ b = 10.0 cm (0.7431)
   b = **7.43 cm**

   If you wanted to check this, see if tan 42° $= \frac{6.69}{7.43}$

   Or see if $(6.69)^2 + (7.43)^2 = (10.0)^2$

END OF LESSON A

LESSON B, page 8

The solutions to the problems in this lesson can often be found in a number of ways, so if you get the correct answer with different formulas, you are probably doing it correctly. Below, we may give alternate solutions or alternate formulas that could be used.

1.′ $v_f = v_i + at$

$= 0 + (5.0\ m/s^2)\ (3.0\ s)$

$\mathbf{v_f = 15\ m/s}$ **(Note why the units end up as m/s)**

$\mathbf{d = v_it + \frac{1}{2}\ at^2}$

$= 0 + \frac{1}{2}\ (5.0\ m/s^2)\ (3.0s)^2$

$= \frac{1}{2}\ (5.0\ m/s^2)\ (9.0s^2)$

$d = 22.5\ m$ (Note that the 'm' is an appropriate unit for the distance)

Other formulas that could also be used are $v_f^2 = v_i^2 + 2ad$ and $d = \left(\frac{v_i + v_f}{2}\right) t$

2. $v_{av} = \frac{v_i + v_f}{2} = \frac{40\ m/s + 60\ m/s}{2} = \mathbf{50\ m/s}$ (regardless of the time)

Alternate method for number 2:

$$d = v_{av}\ t = \left(\frac{40\ m/s + 60\ m/s}{2}\right) 5\ s = 250\ m$$

$$\therefore\ v_{av} = \frac{250\ m}{5.0s} = \mathbf{50\ m/s}\ \text{(for 5.0s)}$$

$$d = v_{av}\ t = \left(\frac{40\ m/s + 60\ m/s}{2}\right) 10\ s = 500\ m$$

$$v_{av} = \frac{500\ m}{10.0s} = \mathbf{50\ m/s}\ \text{(for 10.0s)}$$

LESSON B, page 9

3. $$d = \left(\frac{v_i + v_f}{2}\right) t$$

$$45\ \text{m} = \left(\frac{0 + 30\ \text{m/s}}{2}\right) t \qquad a = \frac{v_f - v_i}{t}$$

$$\therefore\ t = \frac{45\ \text{m}}{15\ \text{m/s}} \qquad a = \frac{30\ \text{m/s} - 0}{3.0\text{s}}$$

$$\therefore\ t = 3.0\text{s} \qquad a = 10\ \text{m/s}^2$$

Other formulas that could be used are:

$v_f^2 = v_i^2 + 2ad$ to get 'a';

then $t = \dfrac{v_f - v_i}{a}$ to get 't'

or $d = v_it + \frac{1}{2}at^2$ could also be used to get 't'

4. $$a = \frac{v_f - v_i}{t} = \frac{24\ \text{m/s} - 12\ \text{m/s}}{3.0\text{s}} = \mathbf{4.0\ m/s^2}$$

$$d = \left(\frac{v_f + v_i}{2}\right)t = \left(\frac{24\ \text{m/s} + 12\text{m/s}}{2}\right) 3.0\text{s} = \mathbf{54\ m}$$

'd' could also be found by using $d = v_it + \frac{1}{2}at^2$ or $v_f^2 = v_i^2 + 2ad$

LESSON B, page 10

5. $$t = \frac{v_f - v_i}{a} = \frac{(40 - 120)\ \text{m/s}}{-10\ \text{m/s}^2}$$

$\therefore\ t = \mathbf{8.0s}$ (Note that $\dfrac{\text{m/s}}{\text{m/s}^2} = \text{m/s} \times \text{s}^2/\text{m} = \text{s}$

$$d = v_{ave}\ t = \left(\frac{120\ \text{m/s} + 40\text{m/s}}{2}\right) 8\ \text{s} = \mathbf{640\ m}$$

For 'd' you could also use $d = v_it + \frac{1}{2}at^2$ or $d = \dfrac{v_f^2 - v_i^2}{2a}$

LESSON B, page 11

6. $v_i = 80.0$ m/s $\quad a_g = 9.8$ m/s^2

$d = 800$ m $\quad v_f = ?$

$$v_f^2 = v_i^2 + 2a_g d$$

$$= (80\text{ m/s})^2 + 2 \times 9.8\text{ m/s}^2 \times 800\text{ m}$$

$$= 6400\text{ m}^2/\text{s}^2 + 15680\text{ m}^2/\text{s}^2$$

$$= 22080\text{ m}^2/\text{s}^2$$

$$v_f = 22080\text{ m/s}$$

$$= 148.6\text{ m/s} = 1.5 \times 10^2\text{ m/s}$$

LESSON B, page 13

1. (a) 100 km/h south

   (b) 0 km/h (Relative to each other, they are at rest.)

   (d) 1 km/h north

2. See the boxed part on page 12

3. No, there is no absolute frame of reference.

LESSON B, page 20

1. This question can have many different answers, depending on the units you use. For length, you could use 'm' or 'km'. For time, you could use 's', or 'min' or 'h'. Using 'km' and 'h'.

$$d = \left(\frac{v_f + v_i}{2}\right)t = \left(\frac{0 + 30\text{ km/h}}{2}\right)\frac{1}{60}\text{ h} = \mathbf{0.25\ km}$$

$$a = \frac{v_f - v_i}{t} = \frac{(30 - 0)\text{km/h}}{1/60\text{ h}} = \mathbf{1.8 \times 10^3\ km/h^2}$$

Using 'm' and 's',

$$30 \text{ km/h} = 30 \text{ km/h} \times \frac{1000 \text{ m}}{\text{km}} \times \frac{1 \text{ h}}{3600\text{s}} = 8.33 \text{ m/s}$$

$$d = \left(\frac{v_f + v_i}{2}\right) t = \left(\frac{0 + 8.33 \text{ m/s}}{2}\right) 60\text{s} = \mathbf{250 \text{ m}}$$

$$a = \frac{v_f - v_i}{t} = \frac{8.33 \text{ m/s} - 0 \text{ m/s}}{60\text{s}} = \mathbf{0.14 \text{ m/s}^2}$$

You could also get 'a' using $a = \frac{v_f - v_i}{t}$, and then you could get 'd' using $v_f^2 = v_i^2 + 2ad$

LESSON B, page 20

2. (a) $a = \frac{v_f - v_i}{t} = \frac{60 \text{ m/s} - 0 \text{ m/s}}{15.0\text{s}} = \mathbf{4.0 \text{ m/s}^2}$

(b) $a = \frac{v_f - v_i}{t} = \frac{60 \text{ m/s} - 60 \text{ m/s}}{4\text{s}} = \mathbf{0 \text{ m/s}^2}$

LESSON B, page 21

(c) The acceleration during the final 12.0 s period.

$$a = \frac{v}{t} = \frac{-24 \text{ m/s} - 60 \text{ m/s}}{12.0 \text{ s}} = \frac{-84 \text{ m/s}}{12 \text{ s}} = -7.0 \text{ m/s}^2$$

Answer: $\mathbf{-7.0 \text{ m/s}^2}$

(d) The displacement of the body from its starting point to its position at the end of the 12.0s period.

Time Interval	Average Velocity	Displacement
First 15s	+30 m/s	30 m/s × 15s = +450 m
Next 4s	+60 m/s	60 m/s × 4.0s = +240 m
Final 12s	$\frac{[60 + (-24)] \text{ m/s}}{2} = +18 \text{ m/s}$	18 m/s × 12s = +216 m
		Total displacement is 906m

(e) The total distance travelled by the body. Notice that the direction of motion changes in the last 12s. So the total distance travelled should be found to be larger than the displacement found in (d).

For the first 19s, the motion is all in the same direction. Thus the distance travelled is 690 m.

In the final 12s, the direction of motion changes after the body comes to a stop.

The time taken to stop is $t = \dfrac{v}{a} = \dfrac{(60 - 0)\text{m/s}}{7.0\ \text{m/s}^2} = 8.6\text{s}$

The body moves $d = v_{av}t = \dfrac{60\ \text{m/s}}{2} \times 8.6\text{s} = 258\ \text{m}$ before stopping.

The time taken to reach -24 m/s is $t = \dfrac{[0 - (-24)]\text{m/s}}{7.0\ \text{m/s}^2} = 3.4\text{s}$

So after stopping, the body moves $d = v_{av}t = \dfrac{24\ \text{m/s}}{2} \times 3.4\text{s} = 41\text{m}$

Thus the total distance travelled is 690 m + 258 m + 41 m + = **989 m**

END OF LESSON B

LESSON C, page 1

1. Vector has magnitude and direction. A scalar has only magnitude.

2. (a) vector
   (b) scalar
   (c) scalar
   (d) vector
   (e) scalar

LESSON C, page 2

17 km west

3.9 m south

50 mm east

8.0 m north

LESSON C, page 7

1. a line 5 cm long, pointing up

2. 60 m west

3. 50 m/s west

LESSON C, page 8

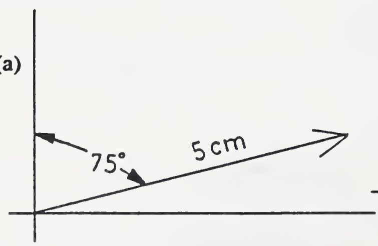
4. If 1 cm = 20 m/s then  (Different scales would result in different lengths of lines)

5. (a) 100 m/s 55° E of N (or 35° N of E)
   50 m/s 40° E of S

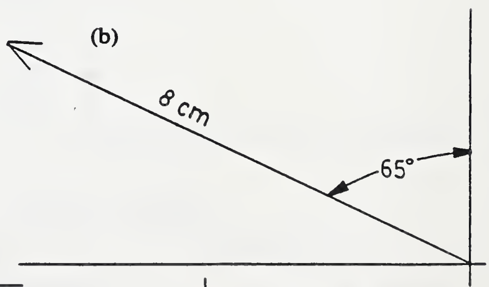
   (b) 240 m 80° W of N
   140 m 70° W of S

LESSON C, page 9

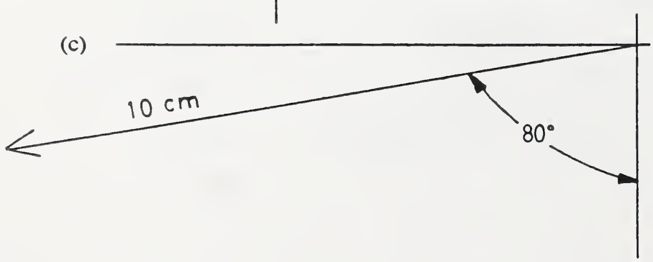
6. (a)

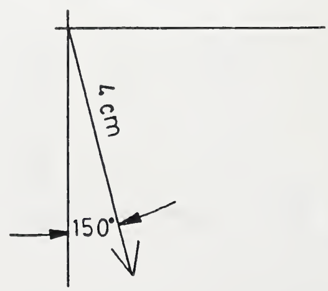


(b)



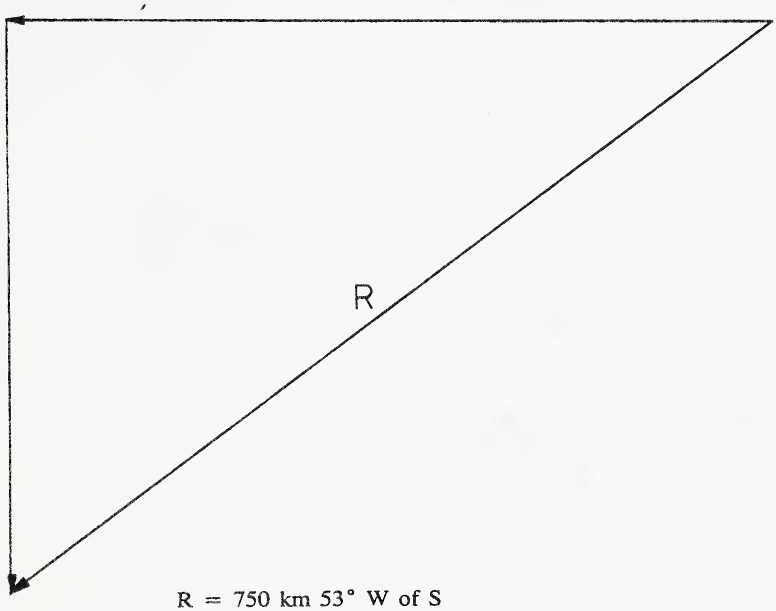
(c)





LESSON C, page 12

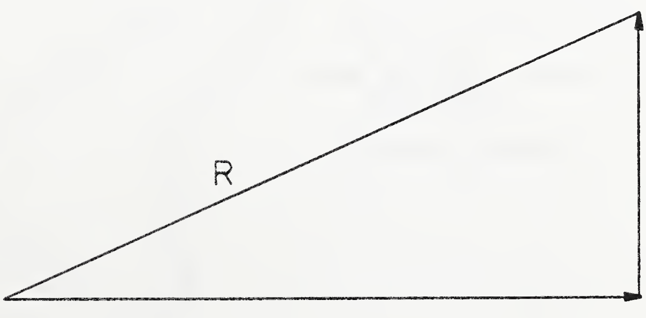
1. Using 1 cm = 50 km



R = 750 km 53° W of S

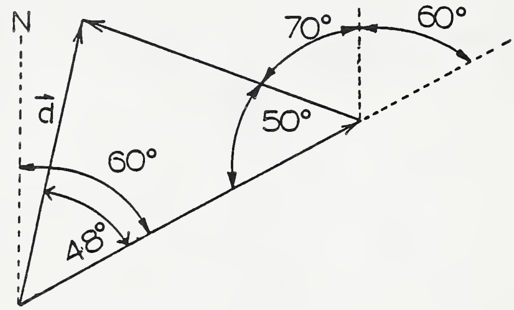
2. Using 1 cm = 40 km/h

R = 440 km/h 66° E of N



LESSON C, page 13

3. A fisherman is near one end of a lake at point A, and he wishes to get from A to point B on the other side. See the diagram. What would be his displacement if he travelled from A to B? Note that the lines drawn in the diagram are not to any particular scale.



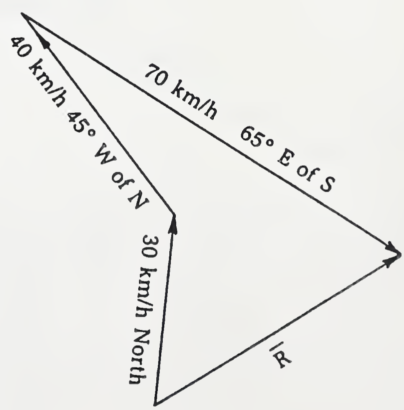
d = 4.6 km at 11.5° E of N

LESSON C, page 16

1. Scale: 1 cm = 10 km/h

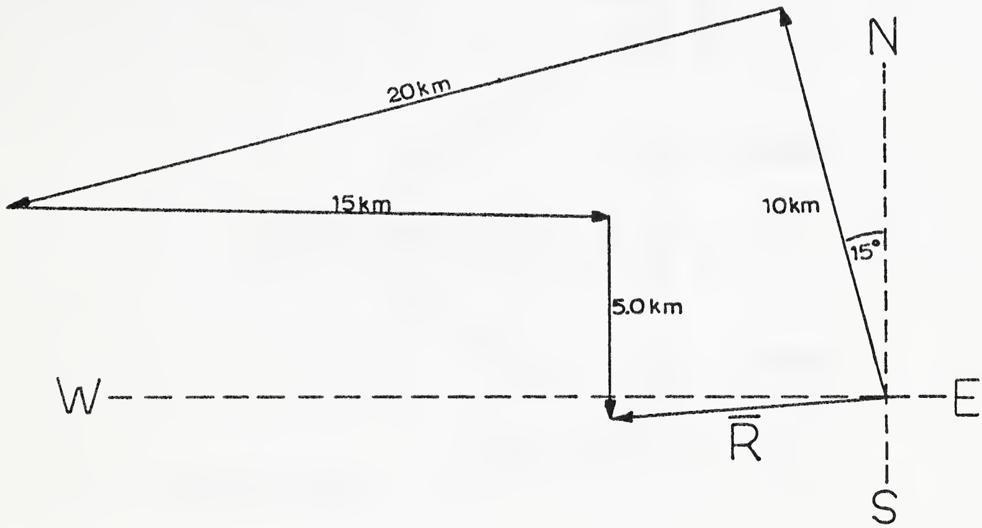
R = 4.5 cm
= 4.5 × 10 km/h = 45 km/h

R = 45 km/h 50° E of N or 40° N of E



LESSON C, page 16

2. (a) 1 cm = 1.25 km



R = 7 km 85° W of S

(b) 7 km 85° E of N

END OF LESSON C

LESSON D, page 1

1. Kinematics is the description of motion. Dynamics explains why motion happened.

2. (a) speed
   (b) distance
   (c) time
   (d) position

LESSON D, page 2

3. (a) See (1), (2), (3) in the middle of page 1.
   (b) You must delete it or revise it as needed.

LESSON D, page 5

1. 5 N in Margaret's favor.

LESSON D, page 6

2. Scale: 1 cm = 10 N

   F_R = 3.5 cm
   = 3.5 × 10 N = 35 N

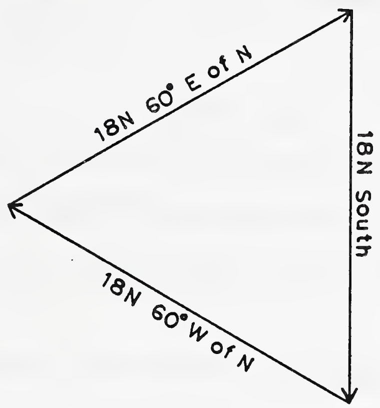
   F_R = 35 N   5° S of W
   or 85° W of S

LESSON D, page 6

3. You start at any one point and draw the three vectors. The tip of the third vector takes you back to your starting position.

   Scale: 1 cm = 3 N

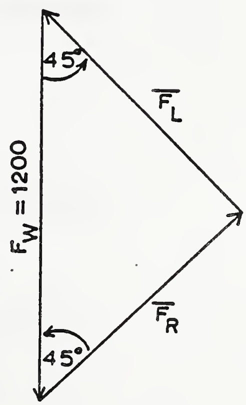
   ∴ F_R = **zero**



LESSON D, page 7

4. Scale: 1 cm = 200 N

   Draw F_w vertically down 6 cm long. On each end draw F_L and F_R at 45° and measure the length of each.



$F_L = F_R = 4.25$ cm
$= 4.25 \times 200$ N $= 850$ N $= \mathbf{8.5 \times 10^2}$ **N**

LESSON D, page 8

1. Inertia is the tendency of a body to remain at rest or to continue moving in a straight line.

LESSON D, page 9

2. first

   second

3. (a) Forces are balanced when the resultant of all forces is zero.

   (b) Unbalanced forces occur when the resultant of the forces on a body is not zero.

4. Once in motion, it has a tendency to remain in motion on its own.

LESSON D, page 10

5. Yes, if the car stops suddenly, the seatbelt will prevent you from hitting the steering wheel or from going through the windshield.

LESSON D, page 11

1. An operational definition describes an activity to explain the meaning of a word.

LESSON D, page 12

2. The greater mass will accelerate the least for a given force and time.
3. Volume is the space occupied by a body. Mass is the measure of inertia of a body.
4. This is very open ended. For example, compare a large cork to a small piece of gold.
5. Give all cartons an equal push. The gold will accelerate the least.
6. Weight is a vector quantity and mass is a scalar quantity. Also, weight varies from place to place but mass stays the same.

END OF LESSON D

LESSON E, page 4

1. (a) The acceleration of a body is proportional to force, and in the direction of the force, but acceleration varies inversely with mass.

   (b) $a = F/m$

2. (a) yes
   (b) 6 N

3. $F = m_a = 1 \text{ kg} \times 15 \text{ m/s}^2 = \mathbf{15\ N}$

   $$a_1 = \frac{15\text{ N}}{3\text{ kg}} = \mathbf{5\ m/s^2}$$

   $$a_2 = \frac{15\text{ N}}{5\text{ kg}} = \mathbf{3\ m/s^2}$$

4. Zero, since acceleration is zero.

5. $m = F/a = \dfrac{10\text{ N}}{4.0\text{ m/s}^2} = \mathbf{2.5\ kg}$

LESSON E, page 5

6. $F = m_a$

   $= 15 \text{ kg} \times 3.0 \text{ m/s}^2$

   $= \mathbf{45\ N}$

Problems

1. $a = \dfrac{(6 - 0)\text{ m/s}}{2\text{ s}} = 3 \text{ m/s}^2 \qquad F_{net} = 11\text{N} - 5\text{N} = 6\text{N}$

   $$m = \frac{F_{net}}{a} = \frac{6\text{N}}{3\text{ m/s}^2} = \mathbf{2\ kg}$$

2. $a = \dfrac{(0 - 15)\text{ m/s}}{0.50\text{ s}} = \mathbf{-30\ m/s^2}$

   $F = ma = 1500 \text{ kg} \times -30 \text{ m/s}^2 = \mathbf{-4.5 \times 10^4 N}$

   The negative sign indicates that the force caused a deceleration.

LESSON E, page 6

3. The engine mass is **not** needed.

$$a = \frac{v_f - v_i}{t} = \frac{(2 - 0)\ \text{m/s}}{5.0\ \text{s}} = \mathbf{0.40\ m/s^2}$$

$$F = ma = 20\ (10\ 000\ \text{kg}) \times 0.40\ \text{m/s}^2$$

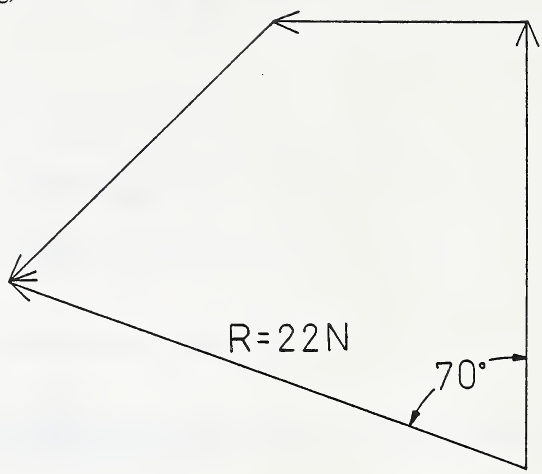
$$= \mathbf{8.0 \times 10^4\ N}$$

4. Using 1 cm = 2 N

$$a = F/m = \frac{22\ \text{N}}{60\ \text{kg}}$$

$$\therefore\ a = 0.37\ \text{m/s}^2$$

at 70° W of N



LESSON E, page 8

1. For every action, there is an equal and opposite reaction.

2. (a) upward force of hand on book
   downward weight of book on hand

   (c) N pole attracts S pole
   S pole attracts N pole

   (d) leaf pushes down on molecules
   molecules push up on leaf

3. (a) No

   (b) Nothing that relies on friction would work.

   He could throw something away from himself opposite the direction in which he wishes to go or he could breathe air in one direction and out in the opposite direction.

LESSON E, page 10

1. (a) $F = ma$
$F = 1.5 \times 10^2 \text{ kg} \times 9.8 \text{ m/s}^2$

$\mathbf{F = 1.5 \times 10^3 \text{ N}}$

(b) $F = ma$
$F = 16.0 \text{ kg} \times 9.8 \text{ m/s}^2$

$\mathbf{F = 1.6 \times 10^2 \text{ N}}$

2. $m = \frac{F}{a} = \frac{1.87 \times 10^3 \text{ N}}{9.8 \text{ m/s}^2} = \mathbf{1.9 \times 10^2 \text{ kg}}$

LESSON E, page 15

1. (a) weight

(b) mass

2. It doubles

3. $F \propto m_1 m_2$

LESSON E, page 16

4. $F \propto \frac{1}{d^2}$

5. the center
6400 km

6. 9600

44N

$\frac{16\,000}{6400} = 2.5$

$\frac{1}{(2.5)^2} \times 100 \text{ N} = 16 \text{ N}$

7. $\frac{1}{(2.5)^2}$

$\frac{4}{(2.5)^2} \times 20 \text{ N} = 13 \text{ N}$

LESSON E, page 17

8. $$\frac{8000\ \text{km}}{6400\ \text{km}}$$

$$\frac{1.25}{1}$$

$$\frac{1}{(1.25)^2}$$

$$\frac{1}{(1.25)^2} \times 50\ \text{N} = 32\ \text{N}$$

9. $$F_2 = F_1 \left(\frac{m_2}{m_1}\right)\left(\frac{d_1}{d_2}\right)^2$$

$$= 6.0 \times 10^6\ \text{N} \left(\frac{1}{10}\right)\left(\frac{6400}{9600}\right)^2$$

$$= \mathbf{2.7 \times 10^5\ N}$$

END OF LESSON E

LESSON F, pages 8 – 12

1. (d)

   $ma_gd = F_wd$

2. (b)

   $P = Fv$

3. (a)

   Only momentum is conserved

4. No. The law is about mass not volume.

   (When the liquids are combined to form a solution the total volume occupied by the liquids is less.)

5. Your answer depends on your mass, m.

   $p = mv = \mathbf{m\ (1.4\ m/s\ S)}$

   If, for example, your mass is 80 kg, then your momentum is 112 kg m/s S.

6. His velocity will be 8 m/s to the right.

   $p = mv$ so

   $$v = \frac{p}{m} = \frac{800 \text{ kg m/s}}{100 \text{ kg}} = 8 \text{ m/s to the right.}$$

7. The speed of the car will be **1.6 m/s**

   $p_1 + p_2 = p_1' + p_2'$

   $(5000 \text{ kg})(2.4 \text{ m/s}) + 0 = (5000 \text{ kg} + 2500 \text{ kg})\, v$

   $$v = \frac{(5000 \text{ kg})(2.4 \text{ m/s})}{7500 \text{ kg}} = 1.6 \text{ m/s}$$

8. (a) It's velocity will be **13.5 m/s**

   Momentum is conserved: $p_1 + p_2 = p_1' + p_2'$

   Let v be the unknown velocity.

$p_1 = 14.0 \text{ kg} \times 14.0 \text{ m/s} = 196 \text{ kg m/s}$

$p_2 = 12.0 \text{ kg} \times 10.0 \text{ m/s} = 120 \text{ kg m/s}$

$p_1 + p_2 = 316 \text{ kg m/s}$

$p_1' = 14.0 \text{ kg} \times 11.0 \text{ m/s} = 154 \text{ kg m/s}$

$p_2' = 12.0 \text{ kg} \times v$

$p_1' + p_2' = 154 \text{ kg m/s} + 12v \text{ kg}$

$316 \text{ kg m/s} = 154 \text{ kg m/s} + 12v \text{ kg}$

$12v = 162 \text{ m/s}$

$v = 13.5 \text{ m/s}$

(b) Total KE before collision is 1972 J
Use '$KE = \frac{1}{2}mv^2$' to find the KE for each ball.
Total KE $= \frac{1}{2} \times 14.0 \text{ kg } (14.0 \text{ m/s})^2 + \frac{1}{2} \times 12.0 \text{ kg } (10.0 \text{ m/s})^2$
$= 1372 \text{ J} + 600 \text{ J} = \mathbf{1972 \text{ J}}$

(c) Total KE after collision is 1941 J
Total KE $= \frac{1}{2} \times 14 \text{ kg } (11.0 \text{ m/s})^2 + \frac{1}{2} \times 12 \text{ kg } (13.5 \text{ m/s})^2$
$= 847 \text{ J} + 1093.5 \text{ J} = \mathbf{1941 \text{ J}}$

(d) inelastic
There is a loss in kinetic energy (1941 J $<$ 1972 J)

(e) The energy may have been used to create permanent dents in the balls. There would also be some conversion of KE to heat due to friction.

9. $W = Fd = 15.0 \text{ N} \times 3.8 \text{ m} = \mathbf{57 \text{ J}}$

10. The mass is 2.4 kg.
$W = F_w d = ma_g d$

$$m = \frac{W}{a_g d} = \frac{75 \text{ J}}{9.8 \text{ m/s} \times 3.2 \text{ m}} = \mathbf{2.4 \text{ kg}}$$

11. The work done is 1.5 J.

$W = Fd = 500\ N \times 0.003\ m = \mathbf{1.5\ J}$

(We ignored KE)

12. The power is 63 W

$P = \frac{W}{t} = \frac{5.0 \times 10^2\ J}{8.0\ s} = \mathbf{63\ W}$

13. The power developed is 74 W

$P = \frac{Fd}{t} = \frac{30\ (15\ kg \times 9.8\ m/s^2)\ 1.0\ m}{60\ s} = \mathbf{74\ W}$

14. The power delivered is 2.50×10^6 W

$P = Fv = 9.00 \times 10^4\ N \times 27.8\ m/s = \mathbf{2.50 \times 10^6\ W}$

15. The energy consumed is 4.8×10^5 J which is equivalent to 133 W·h.

$P = \frac{W}{t}$

$W = Pt = 800\ W \times 600\ s = \mathbf{4.8 \times 10^5\ J}$

$W = Pt = 800\ W \times \frac{1}{6}\ h = \mathbf{133\ W \cdot h}$

LESSON G, pages 8 - 12

1. (a) $\mathbf{KE = 2 \times 10^{-3}\ J}$

$KE = ½(0.001\ kg)(2\ m/s)^2 = 2 \times 10^{-3}\ J$

(b) $\mathbf{KE = 4 \times 10^{-3}\ J}$

$KE = ½(0.002\ kg)(2\ m/s)^2 = 4 \times 10^{-3}\ J$

(c) $\mathbf{KE = 8 \times 10^{-3}\ J}$

$KE = ½(0.001\ kg)(4\ m/s)^2 = 8 \times 10^{-3}\ J$

2. (a) The kinetic energy of the object is **2000 J.**

$KE = ½\ mv^2 = ½\ (10\ kg)(20\ m/s)^2 = 2000\ J$

(b) A force of $\mathbf{6.7 \times 10^2}$ **N** acted on the object.

$$Fd = W = KE$$

$$F = \frac{KE}{d} = \frac{2000\ J}{3.0\ m} = 6.7 \times 10^2\ N$$

3. (a) $F = ma$ so $a = F/m = \dfrac{100\ N}{200\ kg} = 0.5\ m/s^2$

$$d = ½at^2 \text{ (when } v_i = 0\text{) so } t = \frac{2d}{a}$$

$$= \frac{2(36\ m)}{0.50\ m/s^2}$$

$$= \mathbf{12\ s}$$

$$v_f = v_i + at = 0 + 0.5\ m/s^2\ (12\ s) = \mathbf{6\ m/s}$$

(b) $W = Fd = 100\ N \times 36\ m = 3600\ J = 3.6 \times 10^3\ J = KE$

$$KE = ½mv^2 \quad \text{so} \quad v = \frac{2\ KE}{m}$$

$$= \frac{2(3.6 \times 10^3\ J)}{200\ kg}$$

$$= \mathbf{6\ m/s}$$

4. Potential energy is energy due to position or condition whereas kinetic energy is energy due to motion.

5. Here there are many answers possible. Three examples of situations having PE are as follows:

- a stretched elastic
- a boulder on the edge of a cliff
- two magnetic poles separated a small distance apart.

6. (a) The kinetic energy of the sprinter is 4×10^3 J

$$v_f = v_{av} = \frac{100\ m}{10.0\ s} = 10\ m/s$$

$$KE = ½mv^2 = ½\ 80\ kg\ (10m/s)^2 = \mathbf{4 \times 10^3\ J}$$

(b) He would have to climb **5.1 m.**

$PE = ma_g d$

$$d = \frac{PE}{ma_g} = \frac{4 \times 10^3 \text{ J}}{80 \text{ kg} \times 9.8 \text{ m/s}^2} = 5.1 \text{ m}$$

7. Its potential energy is **392 J.**

$PE = ma_g d = 20 \text{ kg} \times 9.8 \text{ m/s}^2 \times 2.0 \text{ m} = 392 \text{ J}$

8. At the top of the ramp its potential energy is **200 J.** Sliding down it can do **200 J** of work.

9. (a) It would take **1.32×10^9 J** to boost the rocket.

$PE = ma_g \ h = 275 \text{ kg} \times 6.86 \text{ m/s}^2 \times 7.00 \times 10^5 \text{ m}$

$= 1.32 \times 10^9 \text{ J}$

(b) The power needed would be 122 kW

$$P = \frac{W}{t} = \frac{1.32 \times 10^9 \text{ J}}{3 \times 3600 \text{ s}} = \mathbf{1.22 \times 10^5 \text{ W}}$$

10. (a) The object must have a speed of **7.9 m/s**

$\frac{1}{2}mv^2 = KE_{bottom} = PE_{top} = ma_g d$

$\frac{1}{2}v^2 = a_g d$

$v = 2 \ a_g d = 2 \times 9.8 \text{ m/s}^2 \times 3.2 \text{ m} = 7.9 \text{ m/s}$

(b) It will have the most PE at the top.

(c) It will have the most KE at the bottom. (as soon as it leaves the bar)

(d) Half-way up it will have half KE and half PE.

(e) At this point its KE is **16 J.**
$PE = ma_g d = 1.0 \text{ kg}(9.8 \text{ m/s}^2)(1.6 \text{ m}) = 15.68 \text{ J}$

LESSON H, pages 10 – 13

1. Heat is a measure of the total kinetic energy of the molecules in a body.

2. momentum

3. KE of the body is converted to heat, which is KE on a small scale.

4. Friction at the pendulum support and air friction cause the pendulum to come to rest. This lost KE is converted to heat.

5. (a) His average acceleration is 0.35 m/s^2.

$$a = \frac{v_f - v_i}{t} = \frac{63.0\ m/s - 0}{180\ s} = \mathbf{0.35\ m/s^2}$$

(b) air friction reduces the acceleration.

(c) Had he fallen freely his kinetic energy would be 1.2×10^8 J.

$$v_f = v_i + at = 0 + (9.8\ m/s^2)(180\ s) = 1764\ m/s$$

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}\ (80.0\ kg)(1764\ m/s)^2 = \mathbf{1.2 \times 10^8\ J}$$

(d) He actually possesses 1.6×10^5 J.

$$KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(80.0\ kg)(63\ m/s)^2 = \mathbf{1.6 \times 10^5\ J}$$

6. The molecules begin to move faster.

7. The molecules begin to move more slowly.

8. Temperature is related to the average KE of the molecules whereas heat is a measure of the total KE of all the molecules of a body.

9. The heat required is $\mathbf{7.5 \times 10^7}$ **J.**

For melting, $h = 25 \text{ kg} \times 3.35 \times 10^5 \text{ J/kg} = 0.8375 \times 10^7 \text{ J}$

For heating, $h = 25 \text{ kg} \times 4190 \text{ J/kg°C} \times 100\text{°C} = 1.0475 \times 10^7 \text{ J}$

For vaporization, $h = 25 \text{ kg} \times 2.261 \times 10^6 \text{ J/kg} = 5.6525 \times 10^7 \text{ J}$

Total is 7.5375×10^7 J

10. (a) See the list on page 8.

(b) Your answers may vary. The following are sample answers:

- solar cell: Light energy is converted to electrical energy.
- electric motor: Electrical energy is converted to mechanical energy.
- battery: chemical energy is converted to electrical energy.

11. The heat released is $\mathbf{3.5 \times 10^3}$ **J.**

For copper $h = (0.18 \text{ kg})(390 \text{ J/kg °C})(20\text{°C}) = 1404 \text{ J}$

For water $h = (0.025 \text{ kg})(4190 \text{ J/kg °C})(20\text{°C}) = 2095 \text{ J}$

Total heat is 3499 J

12. The thermometer would absorb $\mathbf{1.14 \times 10^3}$ **J** of energy.

$t = 90.0\text{°C} - 20.0\text{°C} = 70.0\text{°C}$

For glass $h = (0.0240 \text{ kg})(670 \text{ J/kg°C})(70.0\text{°C}) = 1125.6 \text{ J}$

For mercury $h = (0.00150 \text{ kg})(139 \text{ J/kg°C})(70.0\text{°C}) = 14.595 \text{ J}$

Total heat = 1140.195 J

END OF LESSON H

LESSON I, page 17

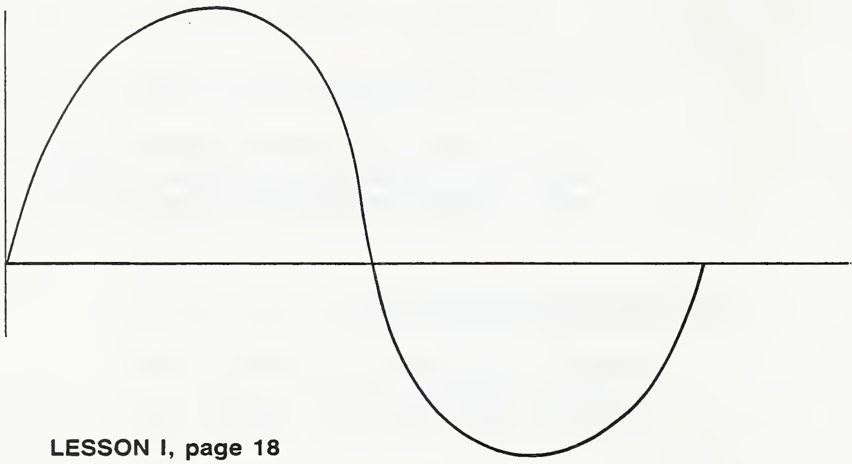
1. A wave is an energy carrying disturbance in a medium which usually has a regular pattern.

2. (a) perpendicular to direction of motion

   (b) 

3. (a) Amplitude is the distance from the level surface of the medium to the peak or crest.

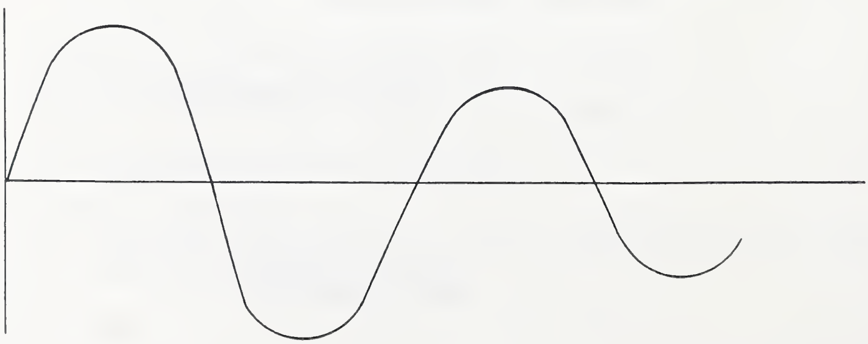
   (b)



LESSON I, page 18

4. (a) Damping is the loss of amplitude or energy as a wave travels out.

   (b)



(c) It goes to heat in the medium. Or it transfers its energy of motion to objects.

5. A longitudinal wave is a wave in which the particles of the medium vibrate parallel to the direction of wave propogation.

6. A compression is a region of increased medium density.

7. Both carry energy and both go out from a source.

8. (a) true

   (b) The wavelength is the distance between successive crests or compressions.

   (c) The period is the time required for a complete vibration.

   (d) The frequency is the number of vibrations per second.

9. $f = \frac{1}{T} = \frac{1}{0.4s} = \mathbf{2.5\ Hz}$

10. $T = \frac{1}{f} = \frac{1}{60/s} = \mathbf{0.017\ s}$

11. (a) $f = \frac{1}{T} = \frac{1}{0.010\ s} = \mathbf{100\ Hz}$

    (b) 12 cm

    (c) 1.0 cm

    (d) 0.010 s

    (e) $v = f\lambda = 100/s \times 12\ cm = \mathbf{1200\ cm/s}$

12. The two points should be half a wavelength or 6 cm apart.

13. $v = f\lambda$

    $= \frac{150}{s} \times 2.0 \times 10^{-2}\ m$

    $= \mathbf{3.0\ m/s}$

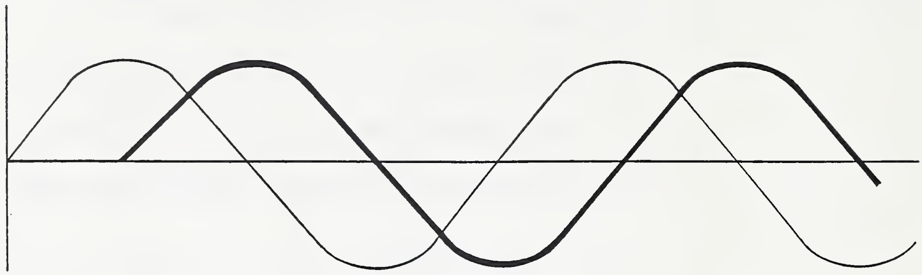
14. $\lambda = \frac{v}{f} = \frac{1.46 \times 10^3 \text{ m/s}}{8.0 \times 10^2 \text{ /s}} = \mathbf{1.8\ m}$

15. $f = \frac{v}{\lambda} = \frac{25 \text{ m/s}}{1.25 \text{ m}} = \mathbf{20\ Hz}$

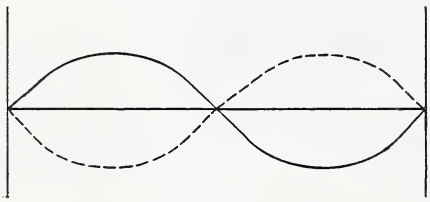
16. (a) frequency and period

(b) velocity and wavelength

17.



18. (a)



(b) The above is 6.5 cm

(c) half of a wavelength

19. A wave front is a line joining points in a wave that are in phase and whose motion began at the same time.

20. A wave ray shows the direction of wave motion.

21. $v = f\lambda = \frac{1.5}{s} \times 4.5 \times 10^{-2}\ m = \mathbf{6.8 \times 10^{-2}\ m/s}$

22. (a) Frequency stays the same since it depends on the source.

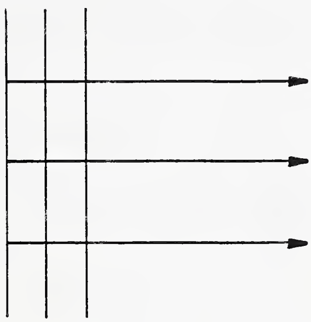
(b) The wavelength is larger since $v = f\lambda$, and if 'v' goes up, then 'λ' goes up.

(c) Period is the same since it depends on the source.

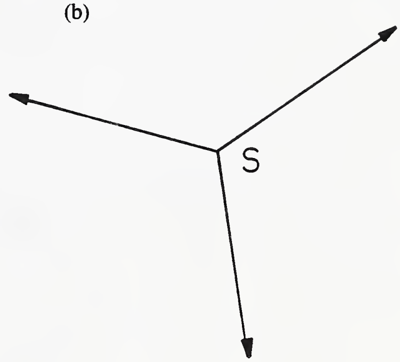
END OF LESSON I

LESSON J, page 17

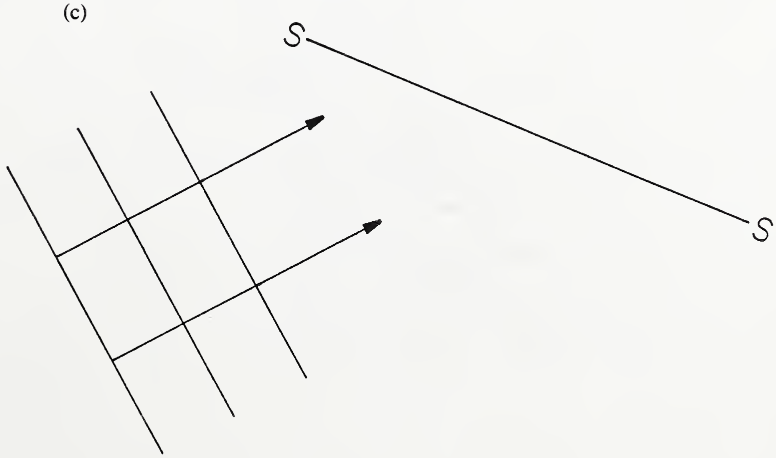
1. (a)



(b)



(c)

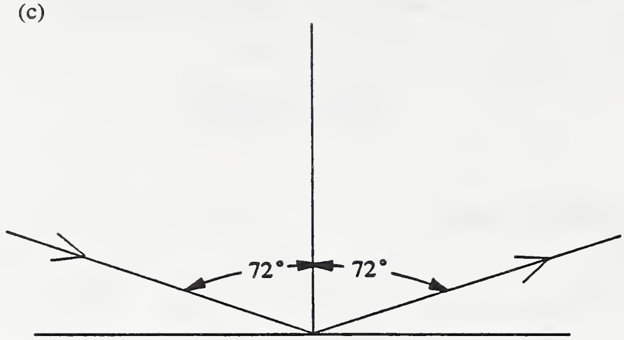


LESSON J, page 18

2. (a) $90° - 18° = 72°$

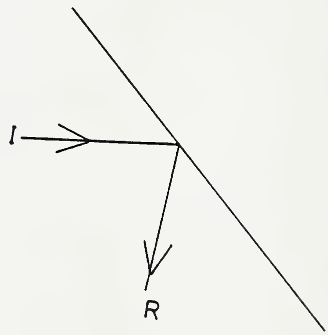
   (b) also $72°$

   (c)

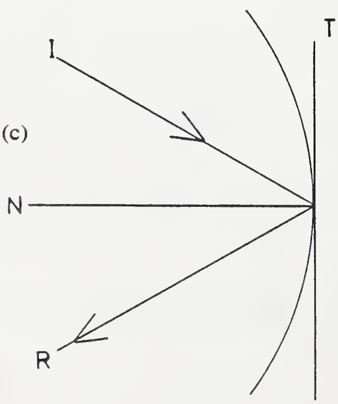


3. (a)

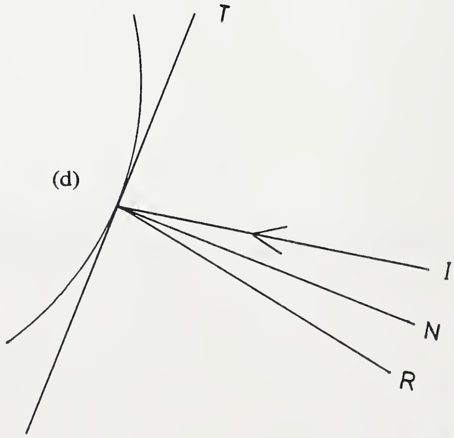
(b)



(c)



(d)



LESSON J, page 19

1. (a) $n = \frac{v_1}{v_2} \therefore v_2 = \frac{v_1}{n} = \frac{30 \text{ cm/s}}{2.5} = \mathbf{12 \text{ cm/s}}$

(b) $n_{12} = \frac{1}{n_{21}}, \therefore n_{21} = \frac{1}{n_{12}} = \frac{1}{2.5} = \mathbf{0.40}$

or $n_{21} = \frac{12 \text{ cm/s}}{30 \text{ cm/s}} = 0.40$

2. $n_{12} = \frac{\sin i}{\sin R}$

$\therefore \sin R = \frac{\sin i}{n_{12}} = \frac{\sin 65°}{2.91} = 0.31144$

$\therefore \mathbf{R = 18°}$

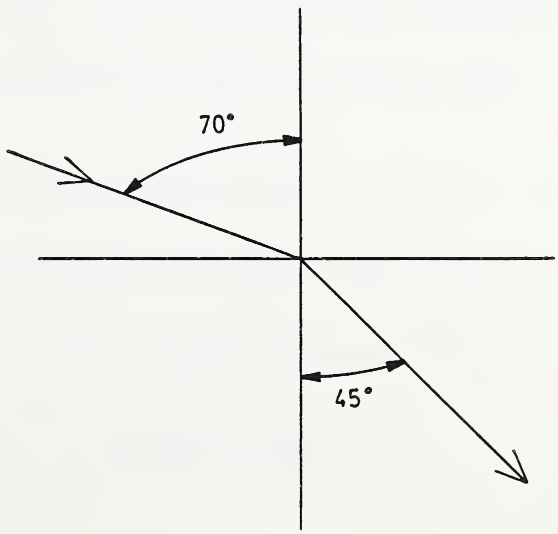
3. (a) $n_{12} = \frac{\sin i}{\sin R}$

$\therefore \sin R = \frac{\sin 70°}{n_{12}} = \frac{0.9397}{1.33} = 0.7065$

$\therefore \mathbf{R = 45°}$

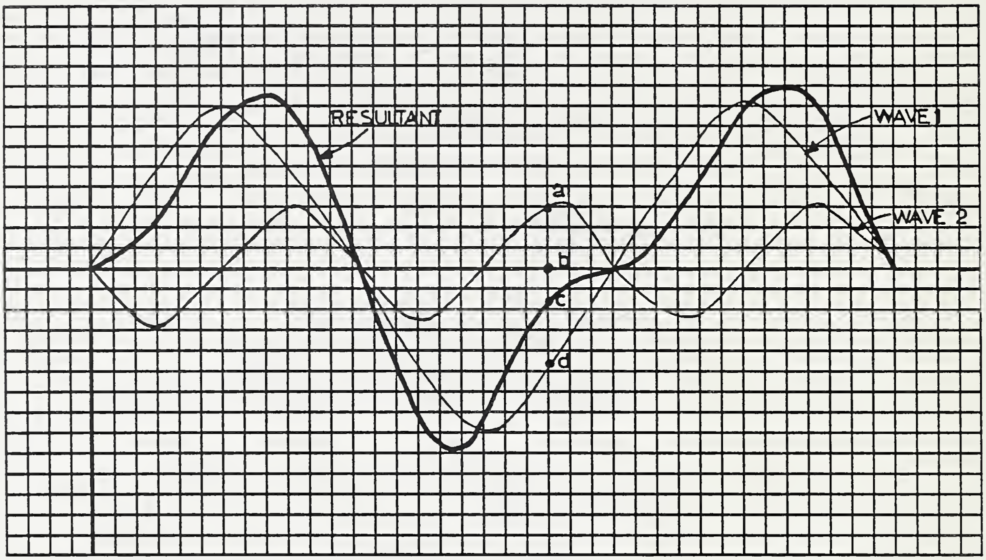
(b) $v_2 = \frac{v_1}{n} = \frac{3.0 \times 10^8 \text{ m/s}}{1.33} = \mathbf{2.3 \times 10^8 \text{ m/s}}$

(c)



LESSON J, page 21

1.



It it easily seen from the above graph that the resultant wave is below wave 1 where wave 2 is below the horizontal axis. Also the resultant wave is above wave 1 where wave 2 is above the horizontal axis. For any position along the horizontal axis the following holds: the resultant wave is as far above wave 1 as wave 2 is above the horizontal axis and the resultant wave is far below wave 1 as wave 2 is below the horizontal axis.

e.g. At position b on the axis the resultant wave is 3 units above wave 1; the distance cd is 3 units. Wave 2 is 3 units above the axis; the distance ab is also 3 units.

2. To diffract, light waves need extremely small openings which are rare.

3. (a) The cork is motionless due to destructive interference.

   (b) The cork goes up and down with maximum amplitude.

END OF LESSON J

PHYSICS UPGRADING
LESSON A

DO NOT send this lesson for correction

SCIENTIFIC NOTATION

The term "scientific notation" is used to indicate a way of writing numbers which can simplify and shorten numerical expressions. For example, a large number such as 1 000 000 000 000 can be written as 1×10^{12}. Similarly, a small number such as 0.000 000 000 001 may be written as 1×10^{-12}. In both these cases **scientific notation** was used in order to write each number more quickly and in a shorter and easier-to-read form.

Scientific notation involves the use of **powers of ten** to express numbers. This is shown by a number of examples in the following table.

Table I

Number	**Number with power of ten**	
1	1×10^0	0
10	1×10^1	1
100	1×10^2	2
1 000	1×10^3	3
10 000	1×10^4	.
1 000 000	1×10^6	.
2 500 000	2.5×10^6	.
2 503	2.503×10^3	

These numbers are known as powers.

Example 1

(a) If you wished to write 7 234 800 in scientific notation you could do it in the following steps:

$$
\begin{aligned}
7\ 234\ 800 &= 7234.8 \times 1000 &&= 7234.8 \times 10^3 \\
&= 7.2348 \times 10^3 \times 1000 &&= 7.2348 \times 10^3 \times 10^3 \\
&= 7.2348 \times 10^6
\end{aligned}
$$

(b) The decimal point was moved <u>six</u> places to the **left**, and the **power** in the final answer was <u>six</u>.

Note that the usual way of writing large numbers in scientific notation is to have one digit to the **left** of the decimal point and to multiply the number by a power of ten in which the **power** is numerically equal to the **number of places** that the decimal point has been moved to the **left**.

Do the following exercises.

QUESTIONS

1. (a) In 721 if the decimal point is to appear between the 7 and the 2, it must be moved __2__ places to the left, and the power in the power of ten is __2__. Thus,

$$721 = 7.21 \times 10^{\_\_}.$$

(b) In 3208 if the decimal point is to appear between the 3 and the 2, it must be moved _____ places to the left, and the power in the power of ten is _____. Thus

$$3208 = 3.208 \times 10^{\_\_}.$$

(c) In 15 917 if the decimal point is to appear between the 1 and the 5, it must be moved _____ places to the left, and the power in the power of ten is _____. Thus,

$$15\ 917 = 1.5917 \times 10^{\_\_}.$$

2. In the following parts of this question, indicate how many places to the left the decimal point must be moved so that it will appear between the first and second digits in the number, and complete the expression of the number in scientific notation by completing the power of ten.

(a) 231 Decimal point moved _____ places to the left.

$$231 = 2.31 \times 10^{\_\_}$$

(b) 5501 Decimal point moved _____ places to the left.

$$5501 = 5.501 \times 10^{\_\_}$$

(c) 93 275 Decimal point moved _____ places to the left.

$$93\ 275 = 9.3275 \times 10^{\_\_}$$

(d) 815 682 Decimal point moved _____ places to the left.

$$815\ 682 = 8.156\ 82 \times 10^{\_\_}$$

3. Express the following numbers in scientific notation.

(a) 153 = __________

(b) 286 = __________

(c) 8592 = __________

(d) 10 158 = __________

(e) 278 643 = __________

(f) 8 953 295 = __________

Small numbers can be reduced to scientific notation by several steps or by moving the decimal point several places to the **right**, and multiplying by a negative power of ten (such as 10^{-1}, 10^{-2}, 10^{-3}, which are the same as $\frac{1}{10}$, $\frac{1}{10^2}$, and $\frac{1}{10^3}$ respectively). See the table below.

Table II

Number in fractional form	Number in decimal form	Number in scientific notation
1	1	1×10^0
$\frac{1}{10}$	0.1	1×10^{-1}
$\frac{1}{100}$	0.01	1×10^{-2}
$\frac{1}{1000}$	0.001	1×10^{-3}
$\frac{1}{10\ 000}$	0.0001	1×10^{-4}
$\frac{1}{1\ 000\ 000}$	0.000 001	1×10^{-6}
$\frac{2}{1000}$	2×0.001	2×10^{-3}
$\frac{756}{100\ 000\ 000}$	$756 \times 0.000\ 000\ 01$	756×10^{-8} or 7.56×10^{-6}

Please note that the negative power is **numerically** equal to the number of places that the decimal point is moved to the **right**. For example, the decimal point is moved 4 places to the right, and the power is -4 when 0.0001 is written in scientific notation: 1×10^{-4}.

$$0.000\ 097\ 231 = 0.000\ 972\ 31 \times \frac{1}{10}$$
$$= 0.000\ 972\ 31 \times 10^{-1}$$
$$= 0.972\ 31 \times \frac{1}{10\ 000}$$
$$= 0.972\ 31 \times \frac{1}{10^4}$$
$$= 0.972\ 31 \times 10^{-4}$$
$$= \frac{9.723\ 1}{100\ 000} = \frac{9.723\ 1}{10^5}$$
$$= 9.723\ 1 \times 10^{-5}$$

Instead of using fractions with powers of ten in the denominator, you could count the number of places the decimal point must be moved to the **right** so that it appears between the first and second digit (between 9 and 7). This number is equal to the **negative** power that should be used in the power of ten if the number is to be expressed in scientific notation.

From the above, the decimal point moved __5__ places to the **right** and 9.723 1 is multiplied by __10^{-5}__ which is a __negative__ power of ten.

Do the following exercise.

QUESTIONS

1. (a) In 0.58, if the decimal point is to appear between the 5 and the 8, it must be moved __1__ place to the right. Thus, the negative power in the power of ten is __-1__, and $0.58 = 5.8 \times 10^{-1}$.

   (b) In 0.013, if the decimal point is to appear between the 1 and the 3, it must be moved _____ places to the right. Thus, the negative power of ten is _____, and 0.013 = 1.3 × 10—.

   (c) In 0.025 6, if the decimal point is to appear between the 2 and the 5, it must be moved _____ places to the right. Thus, the negative power in the power of ten is _____, and 0.025 6 = 2.56 × 10—.

   (d) In 0.008 3, if the decimal point is to appear between the 8 and the 3, it must be moved _____ places to the right. Thus, the negative power in the power of ten is _____, and 0.008 3 = 8.3 × 10—.

   (e) In 0.000 51, if the decimal point is to appear between the 5 and the 1, it must be moved _____ places to the right. Thus, the negative power in the power of ten is _____, and 0.000 51 = 5.1 × 10—.

2. Complete the following table by writing the number of places the decimal point must be moved to the right if it is to appear between the first and second digit, and give the correct negative power to complete the expression of the number in scientific notation. The first one is done for you.

Number	Number of places decimal point moved to the right	Number expressed in scientific notation
0.91	1	9.1×10^{-1}
0.163		$1.63 \times 10^{—}$
0.095		$9.5 \times 10^{—}$
0.008 11		$8.11 \times 10^{—}$
0.000 78		$7.8 \times 10^{—}$
0.000 056		$5.6 \times 10^{—}$

Review of the Rules of Multiplication and Division with Powers

As you should know from your mathematics courses, there are certain rules which must be followed when multiplying or dividing numbers which are expressed using powers. These rules are listed below. Please study them carefully.

1. Two (or more) powers must have the same **base** before the powers can be directly combined (according to Rules 2 and 3 below) in multiplication or division.

   The base of such powers as 10^2, 10^3, 10^{-6}, 10^{-18} is **10.**

   Similarly, the base of such powers as 2^2, 2^5, 2^{-10}, 2^{-3} is **2.**

2. When **multiplying** two (or more) powers of the **same base**, the powers are **added.**

   For example, $10^2 \times 10^3 = 10^{2+3} = 10^5$. This can be shown to be true by the following:

$$\begin{aligned} 10^2 &= 10 \times 10 \\ 10^3 &= 10 \times 10 \times 10 \\ 10^2 \times 10^3 &= (10 \times 10) \times (10 \times 10 \times 10) \\ &= 10 \times 10 \times 10 \times 10 \times 10 \\ &= 10^5 \end{aligned}$$

Other examples:

$$10^3 \times 10^9 \times 10^2 = 10^{3+9+2} = 10^{14}$$

$$10^{-5} \times 10^{-6} = 10^{-5+(-6)} = 10^{-5-6} = 10^{-11}$$

$$10^{-4} \times 10^8 = 10^{-4+8} = 10^4$$

3. When **dividing** two (or more) powers of the **same base**, the powers are **subtracted.**

Examples:

$$\frac{10^5}{10^4} = 10^{5-4} = 10^1 = 10$$

$$\frac{10^8}{10^8} = 10^{8-8} = 10^0 = 1$$

$$\frac{10^2}{10^{12}} = 10^{2-12} = 10^{-10}$$

$$\frac{10^5}{10^{-6}} = 10^{5-(-6)} = 10^{5+6} = 10^{11}$$

Note that $10^1 = 10$, and that $10^0 = 1$. Sometimes when powers are combined, the resulting power will be zero. Any base number raised to a power of 0 equals 1.

Note that when a power is moved from the denominator to the numerator, or vice versa, in a fraction, the sign of the exponent **changes.**
For example,

$$\frac{1}{10^2} = 1 \times 10^{-2}$$

$$10^3 = 1 \times 10^3 = \frac{1}{10^{-3}}$$

$$\frac{10^7}{10^9} = 10^7 \times \frac{1}{10^9} = 10^7 \times 10^{-9} = 10^{-2}$$

To further show this, consider again the fraction $\frac{1}{10^2}$. We have $10^2 = 100$, so that $\frac{1}{10^2} = \frac{1}{100}$. If we complete the division of 100 into 1, we obtain $\frac{1}{100} = 0.01$. Remembering the rule for conversion to scientific notation, in 0.01 the decimal point must be moved two places to the right. This gives a power of -2 and $0.01 = 1 \times 10^{-2}$. Therefore, we have $\frac{1}{10^2} = 1 \times 10^{-2}$.

Do the following questions.

QUESTIONS

1. Can the product (or quotient) of the two numbers 2^5 and 3^7 be found simply by directly combining the powers? Explain your answer.

____

____

2. Find the products of the following numbers by combining the powers.

(a) $2^5 \times 2^2 =$ __________ (d) $10^{-1} \times 10^5 =$ __________

(b) $3^8 \times 3^2 =$ __________ (e) $10^9 \times 10^3 =$ __________

(c) $7^6 \times 7^7 =$ __________ (f) $10^{\frac{1}{2}} \times 10 =$ __________

3. Find the quotients of the following numbers by combining the powers.

(a) $\frac{2^5}{2^2} =$ __________ (d) $\frac{10^{-1}}{10^5} =$ __________

(b) $\frac{3^8}{3^2} =$ __________ (e) $\frac{10^9}{10^3} =$ __________

(c) $\frac{7^6}{7^7} =$ __________ (f) $\frac{10^{\frac{1}{2}}}{10} =$ __________

4. Move the numbers in the denominators of the following fractions to the numerators, and make the appropriate change in the sign of the powers so that the numbers will be equal.

(a) $\frac{1}{10^5} =$ __________ (d) $\frac{2}{10^2} =$ __________

(b) $\frac{1}{10^{-8}} =$ __________ (e) $\frac{5}{10^{-11}} =$ __________

(c) $\frac{1}{2^3} =$ __________ (f) $\frac{3}{10^{80}} =$ __________

Multiplying and Dividing Numbers in Scientific Notation

You may have to multiply and divide numbers which are in scientific notation often when you do physics problems. You can do this fairly easily by using what you know concerning multiplication or division of powers by direct combination of powers. A method of doing such computations is outlined on the following page.

Steps to Complete	Sample Computation
1. Multiply or divide all the powers of ten so that a **single** power of ten results.	$2.0 \times 10^5 \times 6.0 \times 10^2$ $= 2.0 \times 6.0 \times 10^5 \times 10^2$ $= 2.0 \times 6.0 \times 10^7$
2. Multiply or divide the other numbers (the factors of the powers of ten) so that one number results.	$2.0 \times 6.0 \times 10^7$ $= 12 \times 10^7$
3. If necessary, put the number resulting from multiplying or dividing the factors of the powers of ten in scientific notation.	12×10^7 $= (1.2 \times 10^1) \times 10^7$
4. If necessary, combine the two powers of ten to obtain a single number in scientific notation.	$1.2 \times 10^1 \times 10^7$ $= 1.2 \times 10^8$

Do the following questions.

QUESTIONS

1. Find the products of the numbers indicated in the following parts of this question.

   (a) $2 \times 10^2 \times 1 \times 10^5 =$ ______________________

   (b) $7.0 \times 10^8 \times 2.0 \times 10^2 =$ ______________________

   (c) $2.5 \times 10^6 \times 4.0 \times 10^3 =$ ______________________

   (d) $9.0 \times 10^6 \times 9.0 \times 10^{12} =$ ______________________

2. Find the quotients of the numbers indicated in the following parts.

(a) $1.0 \times 10^5 \div (5.0 \times 10^3) =$ ______________________

____

(b) $(9.0 \times 10^2) \div (3.0 \times 10^8) =$ ______________________

____

(c) $(1.6 \times 10^9) \div (4.0 \times 10^9) =$ ______________________

____

(d) $(2.56 \times 10^{17}) \div (1.6 \times 10^{-17}) =$ ______________________

3. Complete the computation indicated below, and give the answer as a single number in scientific notation.

$$\frac{3.00 \times 10^5 \times 6.00 \times 10^{-1} \times 3.00 \times 10^{-2} \times 2.00 \times 10^2}{2.00 \times 10^{-8} \times 6.00 \times 10^{10}}$$

SI (The Latest Version of the Metric System)

The Change to Metric: Through the centuries, many measurement systems have developed, evolving from numerous origins, convenient customs, and local adaptations. Most systems have lacked rational structure. The Imperial system — using the yard, quart, and pound — is one such conglomeration of poorly related units.

About 200 years ago, France decided to bring order out of her chaotic measures and the metric system was born. Although strongly opposed at first, this new system proved effective and gained popularity, so much so, that over 90% of the world's population now lives in countries that have adopted or are changing to the metric system.

Various Versions of Metric Systems: There have been several metric systems, but each new version has added more metric units, causing unnecessary complexity. To make matters worse, in some applications there has been a mixture of both Imperial and metric units, and something had to be done to clean house.

The Latest Version of the Metric System: In 1960 the International System of Units was established as a result of a long series of international discussions. This modernized metric system, called SI, from the French name, **Le Systeme International d'Unites,** is now, as a general world trend, to replace all former systems of measurement, including former versions of the metric system. Canada has decided to convert to SI.

Many European nations are making the change to SI — a change from former metric practice. The United Kingdom, Australia, New Zealand, South Africa, and others are adopting SI, while countries such as India, China, and Japan are updating their metric practice to conform to SI. In the United States, major industries are tooling up for metric conversion, and their choice, too, is SI.

It's SI, not S.I. — omit the periods.

It's just called SI not the "SI system", since the "S" stands for the word "system".

SI is Similar but Different: SI includes familiar metric units such as the metre and kilogram. There are, however, a number of changes from former metric systems. For instance, the centigrade temperature scale is called the Celsius (pronounced sell-see-us) scale when used for general purposes. This is a change in name only, so that 20°C, formerly read as "twenty degrees centigrade", is now read as "twenty degrees Celsius." There is no change in the scale, only in the name. Water still freezes at 0°C and boils at 100°C (degrees Celsius, that is). This kind of change is not difficult for those who are familiar with older metric systems.

Numbers moulded metrics: The metric system was based on the convenience of the decimal number system. Units are related by factors such as 10, 100, and 1000. This makes computation in the metric system much simpler than that with Imperial measures. A great deal of the arithmetic merely involves the shifting of the decimal, without tedious calculations.

This course is written using SI units exclusively. The table of units on the following page is a reference guide so you can easily understand what these units mean.

PLEASE: Try to think SI Metric and use these metric measures whenever possible.

Table of Prefixes

Prefix	Symbol	Meaning	Multiplier	
tera	T	one trillion	1 000 000 000 000	$= 10^{12}$
giga	G	one billion	1 000 000 000	$= 10^{9}$
*mega	M	one million	1 000 000	$= 10^{6}$
*kilo	k	one thousand	1 000	$= 10^{3}$
hecto	h	one hundred	100	$= 10^{2}$
deca	da	ten	10	$= 10^{1}$
*		one	1	$= 10^{0}$
deci	d	one tenth of a	0.1	$= 10^{-1}$
*centi	c	one hundredth of a	0.01	$= 10^{-2}$
*milli	m	one thousandth of a	0.001	$= 10^{-3}$
micro	µ	one millionth of a	0.000 001	$= 10^{-6}$
nano	n	one billionth of a	0.000 000 001	$= 10^{-9}$
pico	p	one trillionth of a	0.000 000 000 001	$= 10^{-12}$
femto	f	one quadrillionth of a	0.000 000 000 000 001	$= 10^{-15}$
atto	a	one quintillionth of a	0.000 000 000 000 000 001	$= 10^{-18}$

* - most commonly used

Unit	Symbol	Meaning	Example
terametre	Tm	10^{12} m	Distance from sun to Saturn = 1.4 Tm
gigametre	Gm	10^{9} m	About 3 times distance from earth to moon.
megametre	Mm	10^{6} m	Distance from Calgary to northern Alberta border.
kilometre	km	10^{3} m	Length of brisk 10 min walk.
metre	m	1 m	Height of 3-drawer filing cabinet.
millimetre	mm	10^{-3} m	Thickness of a dime.
micrometre	µm	10^{-6} m	Size of bacteria.
nanometre	nm	10^{-9} m	Length of oil molecule.
picometre	pm	10^{-12} m	Wavelength of gamma rays.
femtometre	fm	10^{-15} m	Diameter of a proton.
attometre	am	10^{-18} m	???

TABLE OF SI UNITS AND NON-SI UNITS PERMITTED FOR USE WITH SI

The units in the table on pages 12, 13, and 14 are put in the order of most probable frequency of use. So units that would be used frequently are put first, and units less frequently toward the last. It is recommended that the student start thinking **entirely metric**, and **not even think about the Imperial units at all!** The student should use metric measurements in his daily life **exclusively** so it becomes a matter of habit.

QUANTITY	NAME	SYMBOL	NOTES
length	millimetre	mm	
	centimetre	cm	
	metre	m	A Volkswagen is about 4 m long.
	kilometre	km	
area	square centimetre	cm^2	
	square metre	m^2	
volume	cubic centimetre	cm^3	1 cm by 1 cm by 1 cm
	cubic metre	m^3	
	millilitre	mL	1000 mL = 1 L
	litre	L	1 L is the volume of a cube 10 cm by 10 cm by 10 cm.
mass	gram	g	
	kilogram	kg	
	milligram	mg	
	tonne	t	1 t is 1 000 kg, this is about the mass of a Volkswagen.
temperature	degree Celsius	°C	A comfortable room has a temperature of 20°C.
time	hour	h	
	minute	min	
	second	s	
numeric dating	In SI dates are expressed in this <u>exact</u> order: YEAR-MONTH-DAY. The month is not written out. EXAMPLE: handwritten 1978-09-08, typed 1978 09 08		

QUANTITY	NAME	SYMBOL	NOTES
speed	kilometre per hour metres per second metres per hour	km/h m/s m/h	A good highway cruising speed would be 100 km/h.
acceleration	metres per second squared	m/s^2	
gasoline consumption	litres per one hundred kilometres	L/100 km	We are likely to express gas consumption by cars in litres per hundred kilometres, that is, the amount of gasoline it takes for the car to travel 100 km. For instance, a large car may consume gas at 20 L/100 km, while a smaller car may only require about 8 L/100 km. The lower the number of litres, the less gas you consume. This way of stating fuel consumption is prevalent in Europe and other metric areas of the world.
pressure	pascal kilopascal	Pa kPa	The atmospheric pressure is about 100 kPa.
power	watt kilowatt	W kW	A lawnmower motor has a power of about 2 kW.
force	newton	N	The newton is roughly the force required by your hand when supporting 2 golf balls.
electric current	ampere	A	
electric potential, potential difference, electromotive force	volt	V	

QUANTITY	NAME	SYMBOL	NOTES
resistance	ohm	Ω	
rotational frequency	revolutions per minute	min^{-1}	An LP record has a rotation frequency of 33 min^{-1}, also written as r/min.
energy, work	joule	J	
density	kilograms per cubic metre	kg/m^3	
relative density	The terms "specific weight" and "specific gravity" should be replaced by the term "relative density". Mercury has a relative density of 13.6, meaning that it is 13.6 times as dense as water. Water is implied as the reference substance for liquids and solids, and air the reference for gases, unless indicated otherwise.		
capacitance	microfarad farad	μF F	
electric charge	coulomb	C	One coulomb is the charge transported in 1 s by a current of 1 A.

RULES FOR WRITING SI

1. The symbols are always printed in Roman (upright) type, irrespective of the type face used in the rest of the text.

2. Symbols are never pluralized: 45 g (not 45 gs)

3. Never use a period after a symbol, except when the symbol occurs at the end of a sentence. This is done because SI symbols are SYMBOLS they are NOT abbreviations.

   Example: the symbol for kilogram is kg NOT kg.

4. Symbols should usually be used and unit names not mixed with symbols.

   Example: 10 kg (preferred), ten kilograms (accepted), never 10 kilograms.

5. Always use a full space between the quantity and the symbol: 45 g (not 45g)

   Exception: For Celsius temperatures the degree sign occupies the space. 32°C (not 32° C or 32 °C)

6. Use decimals, not fractions: 0.25 g (not 1/4 g) (the decimal is a point on the line in English).

7. A zero is always used before a decimal marker: 0.45 g (not .45 g)

8. Symbols are written in lower case, except when the unit is derived from a proper name:

   m for metre; s for second; but N for newton; A for ampere; degree Celsius °C is the only one to be upper case in both name and symbol.

9. Prefixes are printed in Roman (upright) type without spacing between the prefix and the unit symbol: kg for kilogram, km for kilometre

   Only one prefix is applied at one time to a given unit: megagram or tonne, NOT kilokilogram

10. Use spaces to separate long lines of digits into easily readable blocks of three digits with respect to the decimal marker: 32 453.246 072 5

    Exception: A space is optional with a four-digit number: 1 234 or 1234

11. Multiplication of Units in symbolic form is indicated by a dot at mid-letter height between the symbols.

12. Division of Units in symbolic form is indicated by an oblique stroke between the symbols, <u>or</u> by a negative exponent.

Example 1 Convert the following quantities as required.

(a) 0.45 m = ___?___ cm (b) 0.32 kg = ___?___ g

(c) 3×10^8 m/s = ___?___ km/s (d) 1×10^8 μF = ___?___ F

Solution:

(a) 0.45 m = ___?___ cm

Since metre is to be converted into cm, you have to know the relationship between the two i.e. 1 m = 100 cm

$$0.45 \text{ m} \times \frac{100 \text{ cm}}{1 \text{ m}} = 45 \text{ cm}$$

$$0.45 \text{ m} = 45 \text{ cm}$$

Note that the ratio $\frac{100 \text{ cm}}{1 \text{ m}} = 1$, and therefore it does not change the value of the quantity. The ratios of units are selected so that the unwanted units will be cancelled, and the desired units will remain.

(b) 0.32 kg = ___?___ g

Because 1 kg = 1000 g

$$0.32 \text{ kg} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 320 \text{ g}$$

or $$0.32 \text{ kg} = 320 \text{ g} = 3.2 \times 10^2 \text{ g}$$

(c) 3×10^8 m/s = ___?___ km/s

Because 1 km = 1000 m,

$$3 \times 10^8 \text{ m/s} \times \frac{1 \text{ km}}{1000 \text{ m}} = 3 \times 10^5 \text{ km/s}$$

or $$3 \times 10^8 \text{ m/s} = 3 \times 10^5 \text{ km/s}$$

(d) 1×10^8 μF = ___?___ F

Because 1 μF = 10^{-6} F,

$$1 \times 10^8 \text{ μF} \times \frac{10^{-6} \text{ F}}{1 \text{ μF}} = 1 \times 10^2 \text{ F}$$

or $$1 \times 10^8 \text{ μF} = 1 \times 10^2 \text{ F}$$

Example 2: Convert 30 km/h into m/s.

Solution: You have to convert km into m and h into s. You know that

$$1 \text{ km} = 1000 \text{ m and } 1 \text{ h} = 3600 \text{ s}$$

Therefore, $$\frac{30 \text{ km}}{\text{h}} \times \frac{1000 \text{ m}}{\text{km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 8.3 \text{ m/s}$$

PROBLEMS

Convert the following quantities as required. Show the use of the conversion ratios, as indicated in the first problem.

1. $0.50\ \text{m} = 0.50 \times 10^2\ \text{cm}$ Ratio: $\dfrac{1 \times 10^2\ \text{cm}}{1\ \text{m}}$

$= 5.0 \times 10^1\ \text{cm}$ $0.50\ \text{m} \times \dfrac{1 \times 10^2\ \text{cm}}{1\ \text{m}}$

2. 0.35 kg = g Ratio:

= g

3. 1500 m = km Ratio:

= km

4. 100 mL = L Ratio:

= L

5. 1 000 000 000 Ω = MΩ Ratio:

= MΩ

6. $0.000\ 000\ 001\ \text{F} = 10^{-9} \times 10^6\ \mu\text{F}$ Ratio: $1 \times 10^{-9}\ \text{F} \times \dfrac{10^6\ \mu\text{F}}{\text{F}}$

$= 1 \times 10^{-3}\ \mu\text{F}$

7. $4500\ \text{mm} = 4500 \times 10^{-1}\ \text{cm}$ Ratios: $4500\ \text{mm} \times \dfrac{1\ \text{cm}}{10\ \text{mm}}$

$= 450\ \text{cm}$

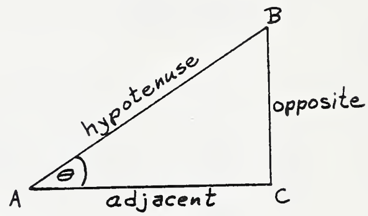
$= 4.5\ \text{m}$ $450\ \text{cm} \times \dfrac{1\ \text{m}}{100\ \text{cm}}$

8. $1.0\ \text{m}^3 =$ cm^3 Ratios: $1.0\ \text{m}^3 \times \left(\dfrac{100\ \text{cm}}{\text{m}}\right)^3$

9. 36 km/h = m/s Ratios:

Trigonometry

Trigonometry means triangle measurement. Although you may never have studied trigonometry, you will find that the principles that will be introduced are not only simple but very useful. The relations we will consider will all be confined to right angled triangles, that is, to triangles in which one angle is 90°. Let ABC be a right angled triangle and let angle A be θ (theta). By definition, the ratio BC/AB is called the sine of θ, the ratio AC/AB is called the cosine of θ and the ratio BC/AC is called the tangent of θ.

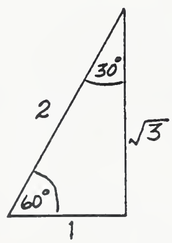


$$\sin\theta = \frac{BC}{AB}$$

$$\cos\theta = \frac{AC}{AB}$$

$$\tan\theta = \frac{BC}{AC}$$

If you have difficulty at first remembering what sine, cosine, or tangent is, you may organize a simple abbreviation for each. From the definition of sin the important words are sin, opposite, and hypotenuse. Take the first letter of each word and you have SOH. For cos you will have CAH and for tangent you will have TOA. In the definition of tan the important words are tan, opposite, and adjacent so you have the abbreviation TOA. Placing the three of them together, you have SOH, CAH, TOA. To get the idea of ratio, study the diagram below.



$$\text{sin of an angle} = \frac{\text{side opposite the angle}}{\text{hypotenuse}}$$

$$\text{cos of an angle} = \frac{\text{side adjacent to the angle}}{\text{hypotenuse}}$$

$$\text{tan of an angle} = \frac{\text{side opposite to the angle}}{\text{side adjacent}}$$

$\sin 60^\circ = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.866$ and $\sin 30^\circ = \frac{1}{2} = 0.5$

$\cos 60^\circ = \frac{1}{2} = 0.5$ and $\cos 30^\circ = \frac{\sqrt{3}}{2} = \frac{1.732}{2} = 0.866$

$\tan 60^\circ = \frac{\sqrt{3}}{1} = 1.732$ and $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{1}{1.732} = 0.577$

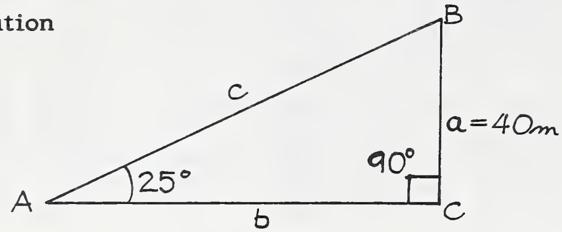
A table of trigonometric functions appears on the next page. For practice vertify some of the values on that table using your calculator.

Angle	Sine	Cosine	Tangent	Angle	Sine	Cosine	Tangent
1°	0.0175	0.9998	0.0175	46°	0.7193	0.6947	1.0355
2°	0.0349	0.9994	0.0349	47°	0.7314	0.6820	1.0724
3°	0.0523	0.9986	0.0524	48°	0.7431	0.6691	1.1106
4°	0.0698	0.9976	0.0699	49°	0.7547	0.6561	1.1504
5°	0.0872	0.9962	0.0875	50°	0.7660	0.6428	1.1918
6°	0.1045	0.9945	0.1051	51°	0.7771	0.6293	1.2349
7°	0.1219	0.9925	0.1228	52°	0.7880	0.6157	1.2799
8°	0.1392	0.9903	0.1405	53°	0.7986	0.6018	1.3270
9°	0.1564	0.9877	0.1584	54°	0.8090	0.5878	1.3764
10°	0.1736	0.9848	0.1763	55°	0.8192	0.5736	1.4281
11°	0.1908	0.9816	0.1944	56°	0.8290	0.5592	1.4826
12°	0.2079	0.9781	0.2126	57°	0.8387	0.5446	1.5399
13°	0.2250	0.9744	0.2309	58°	0.8480	0.5299	1.6003
14°	0.2419	0.9703	0.2493	59°	0.8572	0.5150	1.6643
15°	0.2588	0.9659	0.2679	60°	0.8660	0.5000	1.7321
16°	0.2756	0.9613	0.2867	61°	0.8746	0.4848	1.8040
17°	0.2924	0.9563	0.3057	62°	0.8829	0.4695	1.8807
18°	0.3090	0.9511	0.3249	63°	0.8910	0.4540	1.9626
19°	0.3256	0.9455	0.3443	64°	0.8988	0.4384	2.0503
20°	0.3420	0.9397	0.3640	65°	0.9063	0.4226	2.1445
21°	0.3584	0.9336	0.3839	66°	0.9135	0.4067	2.2460
22°	0.3746	0.9272	0.4040	67°	0.9205	0.3907	2.3559
23°	0.3907	0.9205	0.4245	68°	0.9272	0.3746	2.4751
24°	0.4067	0.9135	0.4452	69°	0.9336	0.3584	2.6051
25°	0.4226	0.9063	0.4663	70°	0.9397	0.3420	2.7475
26°	0.4384	0.8988	0.4877	71°	0.9455	0.3256	2.9042
27°	0.4540	0.8910	0.5095	72°	0.9511	0.3090	3.0777
28°	0.4695	0.8829	0.5317	73°	0.9563	0.2924	3.2709
29°	0.4848	0.8746	0.5543	74°	0.9613	0.2756	3.4874
30°	0.5000	0.8660	0.5774	75°	0.9659	0.2588	3.7321
31°	0.5150	0.8572	0.6009	76°	0.9703	0.2419	4.0108
32°	0.5299	0.8480	0.6249	77°	0.9744	0.2250	4.3315
33°	0.5446	0.8387	0.6494	78°	0.9781	0.2079	4.7046
34°	0.5592	0.8290	0.6745	79°	0.9816	0.1908	5.1446
35°	0.5736	0.8192	0.7002	80°	0.9848	0.1736	5.6713
36°	0.5878	0.8090	0.7265	81°	0.9877	0.1564	6.3138
37°	0.6018	0.7986	0.7536	82°	0.9903	0.1392	7.1154
38°	0.6157	0.7880	0.7813	83°	0.9925	0.1219	8.1443
39°	0.6293	0.7771	0.8098	84°	0.9945	0.1045	9.5144
40°	0.6428	0.7660	0.8391	85°	0.9962	0.0872	11.4301
41°	0.6561	0.7547	0.8693	86°	0.9976	0.0698	14.3007
42°	0.6691	0.7431	0.9004	87°	0.9986	0.0523	19.0811
43°	0.6820	0.7314	0.9325	88°	0.9994	0.0349	28.6363
44°	0.6947	0.7193	0.9657	89°	0.9998	0.0175	57.2900
45°	0.7071	0.7071	1.0000	90°	1.0000	0.0000	

Example 3

In the right angle triangle ABC, angle A is 25° and side a is 40 m. Find sides b and c.

Solution



$A = 25^\circ \quad a = 40\,m$

$B = 180^\circ - (90^\circ + 25^\circ) = 65^\circ$

$b = ?$

Using $\tan B = \frac{b}{a}$

$$b = a \tan B$$
$$= 40\text{ m} \tan 65^\circ$$

From page 19 Table of Natural Trigonometry Functions,

$$\tan 65^\circ = 2.1445$$
$$b = 40\text{ m } (2.1445) = 85.7800\text{ m}$$
$$= 86\text{ m}$$

To find the value of c using trigonometry,

$$\sin 25^\circ = \frac{a}{c} = \frac{40\text{ m}}{c}$$

$$c = \frac{40\text{ m}}{\sin 25^\circ} = \frac{40\text{ m}}{0.4226} = 95\text{ m}$$

Example 4

In the right angle triangle ABC, a is 50 cm and b is 40 cm. Find c and angles A and B.

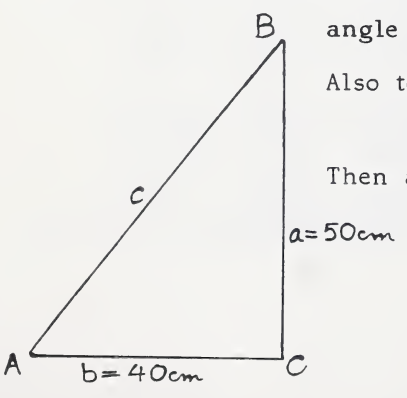
By the Pythagorean theorem

$$c^2 = a^2 + b^2$$
$$= (50)^2 + (40)^2 = 2500 + 1600$$
$$= 4100$$
$$c = \sqrt{4100} = 64\text{ cm}$$

To find angle A,

Use $\tan A = \frac{a}{b}$

$$\tan A = \frac{50\text{ cm}}{40\text{ cm}} = 1.25$$



angle A = 51° (refer to the trigonometry table on page 19)

Also to check, $\sin A = \frac{a}{c} = \frac{50\text{ cm}}{64\text{ cm}} = 0.7813$

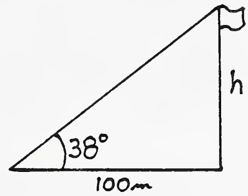
$$A = 51^\circ$$

Then angle $B = 180^\circ - (90^\circ + 51^\circ) = 39^\circ$

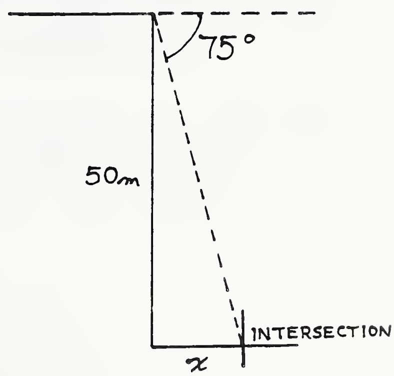
$c = 64$ cm, $A = 51^\circ$, $B = 39^\circ$.

PROBLEMS

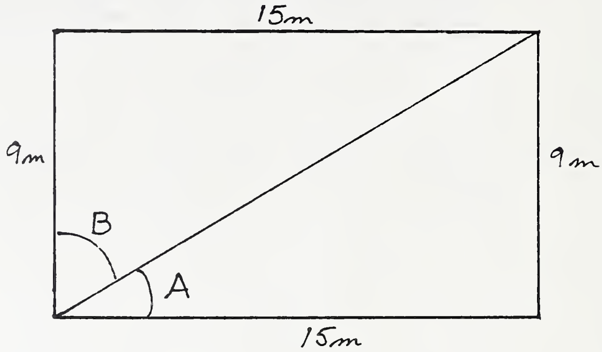
1. A man standing 100 m from the foot of a flagpole, which is at his eye level, observes that the angle of elevation of its top is 38°. Find the height of the pole using trigonometry.



2. From the top of a building 50 m high the angle of depression of the road intersection is 75°. How far from the building is the intersection? (Use trigonometry.)



3. A rectangle is 15 m long and 9.0 m wide. Find the angles which a diagonal makes with the sides. (Use trigonometry.)



4. If one angle of a right angle triangle is 42° and the length of the hypotenuse is 10.0 cm, find the lengths of the other two sides.

Models in Science

Models are usually scale replicas of actual devices. Scientists and engineers use models to help them understand phenomena or devise better devices. James Watt devised a more efficient technique for delivering steam to a piston when he was repairing a working model of a steam engine designed by Thomas Newcomen. Model in this situation implies a working device, smaller than the original, that allows observations and experimentation with a view to improving the larger device. Unfortunately, many natural phenomena cannot be examined in this way because we cannot see them directly e.g. the structure of atom and the propagation of light.

We often use analogies to help us describe and explain reality. Analogies are not real things, but they help us to understand. Scientists' models are like analogies. Scientists use analogies in the form of models or theories to assist them in understanding the world, especially parts of it that they cannot observe directly.

Theories or theoretical models are used:

- to explain the known qualities or properties of a phenomenon;
- to predict properties and behavior that may not be directly observable; and
- to help devise new applications for a known scientific phenomenon.

Once a scientist has what he believes to be a good model, he devises an experiment with which to test it. The experiment may support the theory or it may suggest modification of the theory or may even prove the theory to be incorrect.

Thus scientists and technologists have two types of models to help them understand the world around them. Working models are used if the phenomenon can be observed and theoretical models are used if the phenomenon cannot be observed directly.

You should keep in mind that the scientific use of models is to help us understand the world and that the models must prove themselves able to stand the test of new evidence that becomes available. If they cannot do that, they are either discarded or revised. If you have read this section, then write 'yes' in the space provided. ___________

Lesson Summary

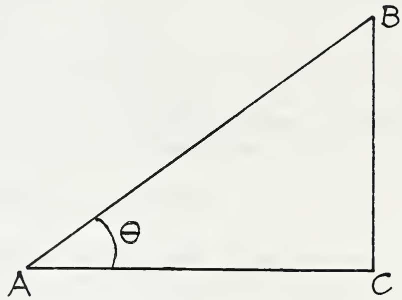
Note that the usual way of writing large numbers in scientific notation is to have one digit to the left of the decimal point and to multiply the number by a power of ten in which the **power** is numerically equal to the **number of places** that the decimal point has been moved to the left. If the decimal point is moved to the right, the power of ten is negative. When multiplying or dividing, powers must have the same base. The powers are added if you are multiplying and subtracted if you are dividing. SI units are the only units in use now in this course.

In a right angled triangle **ABC**,

$\sin\theta = \frac{BC}{AB}$

$\cos\theta = \frac{AC}{AB}$

$\tan\theta = \frac{BC}{AC}$



PHYSICS UPGRADING
LESSON B

DO NOT send this lesson for correction

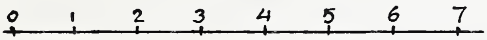
DESCRIPTION OF MOTION

One Dimensional Motion

Perhaps the simplest motion that we can consider is that of motion along a line. For example, we could consider motion along a line that extends from the left to the right across a page, as shown in part (**a**) of Fig. A. Or, we would consider motion along a line that extends up and down (from the top to the bottom of the page) as in part (**b**) of Fig. A. In either case, we would be dealing with **one dimensional motion** because any motion would be exclusively along a single straight line (along a single spatial dimension). In general, motions are not one dimensional. Most of the motions that we observe and are familiar with are three dimensional: they involve movements that are up and down, left-right, and forward-backward. However, in order for us to learn to deal with the study of motion in physics, we should begin with a simpler case. That is why we shall start by considering one-dimensional motion.

Fig. A

(**a**)

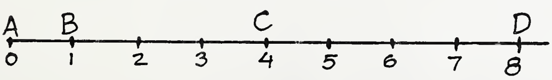


(**b**)

Position, Distance and Speed

The position of a body on a line can be found only in terms of some reference point on a line. Referring to Fig. B, a body might be said to be at point **A** on the line. We could call point **A** the position of the body. If the body then moved to position B, then one way of specifying part of its motion would be to give the distance that it moved from **A** to **B**. In Fig. B, the distance between **A** and **B** is 1 cm. Hence, we could say that in moving from **A** to **B**, the body moved a distance of 1 cm. Similarly, in moving from **A** to C and from **A** to D, the distances travelled would be 4 cm and 8 cm, respectively. What we have done is to take position **A** as a reference point, and assign it a value of 0 cm. Then the determination of positions B, C, and D just involved the measurements of the distances of the points B, C, and D from point **A**.

Fig. B



In order to find the average **speed** of the body in moving from one point to another on the line we need to know the distance as well as the time involved in moving that distance. Average speed can be defined as follows:

$$\text{average speed} = \frac{\text{distance}}{\text{time to travel the distance}}$$

Using a formula:

$$v_{av} = \frac{d}{t}$$

where v_{av} = average speed
d = distance
t = time to travel the distance

Suppose a body moves from point **A** to point **B** in 1 s (see Fig. B). The average speed in travelling from **A** to **B** would be found as

$$v_{av} = \frac{d}{t} = \frac{1\text{ cm}}{1\text{ s}} = 1\text{ cm/s}$$

If we also found that the body moved from point **A** to point C in 2 s, the average speed in travelling from **A** to C would be found as

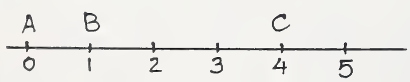
$$v_{av} = \frac{d}{t} = \frac{4\text{ cm}}{2\text{ s}} = 2\text{ cm/s}$$

Instantaneous Speed

If you have ever watched the speedometer in a car or truck, you know that the speed of a vehicle can change quickly from one instant to another. The speed of a body at a particular instant of time is the **instantaneous speed** of the body. For a body which has changes in speed, its instantaneous speed and its average speed are not the same. However, for a body which has a constant speed (a speed that does not change) when moving from one point to another, the instantaneous speed and average speed between those two points are equal.

Referring back to Fig. B, we can assign values for instantaneous speed at each of the points **A**, **B**, and C. This has been shown in Fig. C. We can also specify the times at which these instantaneous speeds occurred. If we set the initial time as being **0** at position **A**, then the time at **B** is 1 s and the time at C is 2 s.

Fig. C



At A, $v_A = 0$, $t_A = 0$

At B, $v_B = 2$ cm/s, $t_B = 1$ s

At C, $v_C = 4$ cm/s, $t_C = 2$ s

The instantaneous speed and average speed are related by the following:

$$v_{ave} = \frac{\text{initial instantaneous speed + final instantaneous speed}}{2}$$

$$v_{ave} = \frac{v_i + v_f}{2}$$

where v_i = initial instantaneous speed
v_f = final instantaneous speed.

To find the average speed between points A and B, we have

$$v_{av} = \frac{v_i + v_f}{2} = \frac{v_A + v_B}{2} = \frac{0 + 2\ \text{cm/s}}{2}$$
$$= 1\ \text{cm/s}$$

For the average speed between positions A and C:

$$v_{av} = \frac{v_i + v_f}{2} = \frac{v_A + v_C}{2} = \frac{0 + 4\ \text{cm/s}}{2}$$
$$= 2\ \text{cm/s}$$

Note that these values are the same as those found earlier.

Rate of Change of Speed

Another important feature of the description of motion of a body is its rate of change of speed. This can be found by finding the amount that the speed changes for a unit length of time (often the unit of time is the second). This can be expressed in formula form as shown below:

$$\text{rate of change of speed} = \frac{\text{change in speed}}{\text{time during which speed changes}}$$

$$a = \frac{v_f - v_i}{t}$$

where a = rate of change of speed
v_f = final speed
v_i = initial speed
t = time during which the instantaneous speed has changed from v_i to v_f

Sometimes the rate of change of speed, a, is called acceleration. However, in a strict sense, acceleration involves more than just a change in speed, as we will see later.

Using our earlier example from Fig. B and C, we can calculate the rate of change of speed as shown below:

From position **A** to position **B**:

$$a = \frac{v_f - v_i}{t} = \frac{v_B - v_A}{t_B - t_A} = \frac{2\ \text{cm/s} - 0}{1\ \text{s}}$$

$$= \frac{2\ \text{cm/s}}{\text{s}} = 2\ \text{cm/s} \times 1/\text{s} = 2\ \text{cm/s}^2$$

From position **A** to position **C**:

$$a = \frac{v_f - v_i}{t} = \frac{v_C - v_A}{t_C - t_A} = \frac{4\ \text{cm/s} - 0}{2\ \text{s}}$$

$$= \frac{2\ \text{cm/s}}{\text{s}} = 2\ \text{cm/s}^2$$

There are two features worth pointing out.

1. The rate of change of speed in this example is unchanging. It is $2\ \text{cm/s}^2$ in both cases. This suggests that the value is constant, or that the rate of change of speed is **uniform.** In most situations which we shall study, the rate of change of speed will be assumed to be uniform.

2. The units of the rate of change of speed can be expressed in terms of $\frac{\text{units of speed}}{\text{units of time}}$ or $\frac{\text{units of distance}}{(\text{units of time})^2}$.

These two ways result in the two expressions $a = \frac{2\ \text{cm/s}}{\text{s}}$ and $a = 2\ \text{cm/s}^2$ which are equivalent.

Summary of Equations of Motion

We have seen three of the formulas or equations which help us to do calculations so that we can describe the motion of a body in one dimension. On page 5, the three equations we have learned about, as well as others, are given.

1. $v_{av} = \frac{d}{t} = \frac{v_i + v_f}{2}$

For a body moving with uniform rate of change of speed (**a** = constant):

2. $a = \frac{v_f - v_i}{t}$ or $v_f = v_i + at$

3. $d = \frac{v_i + v_f}{2} t$

4. $d = v_i t + \frac{1}{2}at^2$

In these equations **v** is speed, **d** is distance, t is time, **a** is rate of change of speed, v_i is the initial speed and v_f is the final speed.

Note: A freely falling body is a body falling under conditions in which air resistance is zero or is so small that it can be ignored. A freely falling body has a rate of change of speed of 9.8 m/s². It is denoted by a_g. Therefore for a freely falling body, equation 4 becomes

$$d = v_i t + \frac{1}{2}a_g t^2$$

Also there is another very important equation, which is not mentioned in some textbooks:

5. $v_f^2 = v_i^2 + 2\,ad$

The method of derivation of these equations is not very important for you. What is important is that you use these equations in problem solving. The more problems you solve the better you will understand them. Work through the following examples.

EXAMPLE 1

The initial speed of a body moving with a uniform rate of change of speed of 5.0 m/s² is 10.0 m/s. Find the speed after 15 s.

Given: $v_i = 10.0$ m/s
$a = 5.0$ m/s²
$t = 15$ s

To find: v_f

Solution:
$$\begin{aligned} v_f &= v_i + at \\ &= 10.0\ \text{m/s} + (5.0\ \text{m/s}^2 \times 15\ \text{s}) \\ &= 10.0\ \text{m/s} + 75\ \text{m/s} = 85\ \text{m/s} \end{aligned}$$

EXAMPLE 2

The initial speed of a body moving with a uniform rate of change of speed of 3.5 m/s^2 is 12 m/s. Find the distance travelled by the body in 0.50 min.

Given: $v_i = 12\ m/s$
$a = 3.5\ m/s^2$
$t = 0.50\ min = 30\ s$

To find: d

Solution:
$$\begin{aligned} d &= v_i t + \tfrac{1}{2}at^2 \\ &= (12\ m/s \times 30\ s) + \left[\tfrac{1}{2} \times 3.5\ m/s^2 \times (30\ s)^2\right] \\ &= 360\ m + (\tfrac{1}{2} \times 3.5\ m/s^2 \times 900\ s^2) \\ &= 360\ m + 1575\ m \\ &= 1935\ m \\ &= 1.9 \times 10^3\ m \end{aligned}$$

EXAMPLE 3

A train starting from rest at a station uniformly gains speed, until, after 2 min, it acquires a speed of 100 km/h. What is the distance travelled by the train during this time interval?

Given: $v_i = 0$
$v_f = 100\ km/h$
$t = 2\ min = 1/30\ h$

To find: d

Solution:
$$\begin{aligned} d &= \frac{v_i + v_f}{2}\, t \\ &= \frac{(0 + 100)\ km/h}{2}\,(1/30\ h) \\ &= \frac{100}{60}\ km = 1.7\ km = 2\ km \end{aligned}$$

EXAMPLE 4

A stone is dropped from a height of 980 m. How long does it take to reach the ground and what is its speed when it reaches the ground?

Given: $v_i = 0$

$d = 980\ \text{m}$

$a_g = 9.8\ \text{m/s}^2$

To find: t and v_f

Solution: $d = v_i t + \frac{1}{2} a_g t^2$

$$980\ \text{m} = 0 + \tfrac{1}{2} \times 9.8\ \text{m/s}^2 \times t^2$$

$$4.9\ t^2 = 980\ \text{s}^2$$

$$t^2 = 200\ \text{s}^2$$

$$t = \sqrt{200}\ \text{s}$$

$$= 14\ \text{s}$$

$$v_f = v_i + at$$

$$= 0 + 9.8\ \text{m/s}^2 \times 14\ \text{s} = 137.2\ \text{m/s}$$

$$= 1.4 \times 10^2\ \text{m/s}$$

Do the following problems.

PROBLEMS

1. A toy car accelerates by means of a small rocket-type engine. If the rate of change of speed during the burn is 5.0 m/s^2 and the burn lasts 3.0 s, determine the speed and the distance travelled at the end of the burn.

2. A car travelling at 40 m/s increases its speed uniformly to 60 m/s. Determine the average speed if the time required was 5.0 s. What would be the average speed if the time required was 10 s?

   Hint: You do not need to find the acceleration.

3. A body starting from rest changes its speed uniformly and attains a speed of 30 m/s after travelling a distance of 45 m. Find the rate of change of speed of the body and the time required to travel this distance.

4. A body moving initially with a speed of 12 m/s changes speed uniformly and attains a speed of 24 m/s in 3.0 s. Find the rate of change of speed of the body and the distance travelled by it in this time interval.

5. The initial speed of a body is 120 m/s and the final speed is 40 m/s. If the rate of change of speed is $^{-}10\ m/s^2$, find the time required to attain this speed and find the distance travelled in this time.

6. A balloon is falling at the rate of 80.0 m/s and when it is at a height of 8.0×10^2 m, a stone is dropped from it. With what speed does the stone hit the ground? (Hint: The stone does not begin to fall from rest. It will have the same initial speed as the balloon, that is, 80.0 m/s.) (Hint: use $v_f^2 = v_i^2 + 2ad$)

Motion and Direction

So far in dealing with motion, we have not mentioned a very important feature: **direction.** If we include the feature of direction we must add to the ideas of motion that we studied earlier (distance, speed, rate of change of speed). As a starting point, study the following statement.

> An object is defined as being in motion with respect to an observer if a line joining the object to the observer changes in either length or direction.

To help in understanding how this allows a person to describe motion, we can think of what happens when two objects are connected together by something like a line, such as a string. For example, suppose you have a weight on a string and you hold the weight and string in your hand. We can think of the string joining the weight (the object) to you (the observer) as being similar to the line. While you hold the string and weight in your hand, the string does not change in length or direction, so you can say that the weight is **at rest** with respect to you. Next, suppose you drop the weight, letting the string slip through your fingers until about 50 cm of string has been released. Then, you tighten your grasp on the string and the fall of the weight stops. While it was falling, the weight was **in motion downward** with respect to you. The increasing length of the string as the weight falls would be similar to the increasing length of a line between you and the weight. In addition, the change in length occurs in a downward direction so that you can describe the motion as being downward.

Now, suppose you hold the length of the string constant at 50 cm, and you twirl the weight in a circle above your head. With respect to your hand, the length of the line between your hand and the weight does not change. However, **direction** is changing continuously as the weight whirls around. Thus, even though the length of the line does not change, since its direction is changing, the weight must be considered to be in motion with respect to you.

When it is said that an object is in motion **with** respect to an observer, the observer is considered to be a reference point which is at rest. This is done in order to make it easier to describe the motion; it is not a way of saying that the observer is not moving. It is just a convenient way of describing motion.

At one time it was believed that it was possible to find a point that was perfectly at rest (or absolutely at rest). It was believed that there was something called the aether (pronounced "eether") which was a kind of "non-material substance" which filled space and which was a fixed frame of reference throughout space. If a body were absolutely at rest, this meant that it did not move with respect to the aether. However, eventually through experiments with light, the existence of the aether was brought into doubt, and the idea was discarded. This also resulted in elimination of the idea that a body could be absolutely at rest — since it was necessary to have a universal frame of reference in order to define the idea of absolute rest.

The idea of the aether is useful because it demonstrates a very important point; motion must be defined with respect to something, such as the centre of the earth, or the surface of the earth, or the sun. However, you cannot say that your point of reference is absolutely at rest or fixed. You can say only that you have made your observations from a particular point, which you find convenient to use. If you used some other point as your reference point, then you might be able to observe the motion of your previous reference point while pretending that your new point is at rest.

Answer the following questions.

QUESTIONS

1. Describe the motion of each of the following objects relative to a car moving north at 100 km/h with respect to the road.

   (a) A hitchhiker standing beside the road.

   ____

   (b) Another car heading north at 100 km/h.

   ____

   (c) An airplane overhead flying south at 300 km/h.

   400 km/h south (relative to the car)

   (d) A car heading north at 101 km/h.

   ____

2. How is motion with respect to an observer defined?

   ____

   ____

   ____

   ____

3. Is it possible to define the idea of absolute rest physically? Why or why not?

   ____

   ____

   ____

   ____

One Dimensional Motion and Direction

In order to see how the concept of direction can be added to our previous ideas of motion we will consider the case of one dimensional motion again. See Fig. D. The straight line in Fig. D is divided into 1 cm segments. Near the centre of the line a point has been labelled **0**. On the right side of the 0 point, there are points labelled +1, +2, +3, and +4 at 1 cm intervals. On the left side of the 0 point there are points labelled −1, −2, −3, and −4 at 1 cm intervals.

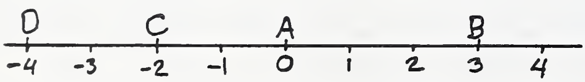
Fig. D

What we are doing here is defining a **direction convention:** points to the right of the 0 point have positive signs, and points to the left of the 0 point have negative signs.

You may have realized that the point "labels" in Fig. D are actually the distances of the points from point 0. By giving positive and negative signs to those distances we have indicated the directions (left or right) of the points along the line from point 0. This gives us a way of specifying motion using both distance and direction.

In Fig. E, the positions of a body along a straight line are indicated by the letters **A**, **B**, **C**, and **D**. We can specify the positions of the body with respect to position **A** by giving their **displacements**. A displacement may be defined as the change in position of a body. Displacement includes the ideas of the size of the change of position and the direction of the change of position. The size of the change of position can be expressed in terms of the length of the line between two points. The direction of the change of position can be expressed in terms of a positive or negative sign, to show that the change of position is to the right or left respectively of the reference point. For example, referring to Fig. E, we can give the displacements of a body at points B, C, and D with respect to point **A** as follows:

Fig. E



Displacement at B with respect to A = +3 cm = $\overline{d}_{BA}$

Displacement at C with respect to A = −2 cm = $\overline{d}_{CA}$

Displacement at D with respect to A = −4 cm = $\overline{d}_{DA}$

Notice that we have given the symbols representing the displacements as well as the values. The d's with bars over them (sometimes small arrows are used instead of bars) represent the displacements. The bars over the d's provide a way for us to distinguish between distances and displacements when we use symbols.

In finding the displacements with respect to point A, we found the distances between each point and point A in centimetres, and we gave a + or - sign to show that the point was to the right or left, respectively, of point A.

If the reference point is changed, then we use the same procedure. Suppose that we take point B as the reference point. Displacements at positions A, C, and D with respect to point B are given below:

$$\overline{d}_{AB} = -3 \text{ cm}$$
$$\overline{d}_{CB} = -5 \text{ cm}$$
$$\overline{d}_{DB} = -7 \text{ cm}$$

Since points A, C, and D are all to the left of point B, the signs of the displacements are all negative. We can consider the other extreme by finding the displacements of points A, C, and B with respect to point D.

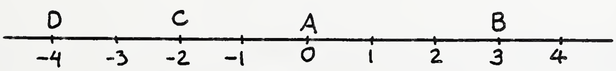
$$\overline{d}_{AD} = +4 \text{ cm}$$
$$\overline{d}_{BD} = +7 \text{ cm}$$
$$\overline{d}_{CD} = +2 \text{ cm}$$

Since points A, B, and C are all to the right of point D, the signs of the displacements are all positive.

Distance and Displacement

It is important to distinguish between distance and displacement. As you have probably realized, the distance travelled by a body is not the same as displacement. One difference is that displacement includes direction and distance does not. Another difference can be understood if we look at the definitions. The distance travelled by a body between two points may be defined as the length of the path travelled by the body in going from one point to another. If a body travels from point A to point D, going through B and C, then the distance travelled by the body is the sum of the distance from A to B, the distance from B to C, and the distance from C to D. Referring to Fig. F, distance AB = 3 cm, distance BC = 5 cm, distance CD = 2 cm, and the distance travelled by a body in going from A to D through B and C is 3 cm + 5 cm + 2 cm = 10 cm. Hence, the distance travelled by the body depends upon the path that the body takes. The displacement at D with respect to A is given by $d_{DA} = -4$ cm. Displacement is not necessarily dependent on the path taken by the body.

Fig. F



Velocity and Acceleration

When we include direction, we no longer deal with just speed or the rate of change of speed. Instead we must use the ideas of **velocity** and **acceleration**. Velocity may be defined as the rate of change of displacement. For average velocity we have

$$\overline{v}_{av} = \frac{\overline{d}}{t}$$

where $\overline{v}_{av}$ = average velocity
$\overline{d}$ = the displacement
t = the time during which the displacement occurs.

Instantaneous velocity is the velocity of a body at some instant of time.

Remember that velocity includes direction. The direction convention that we shall use is the same as that introduced earlier: velocity to the right is positive; velocity to the left is negative.

Some of the situations with which we shall be concerned will involve vertical motion rather than left-right motion. In those cases, we shall adopt the following convention: upward displacements and velocities are positive; downward displacements and velocities are negative.

Acceleration may be defined as the rate of change of velocity. Because it includes direction, acceleration is not the same as the rate of change of speed. Again, the same direction conventions will be used for acceleration as for velocity and displacement:

1. Left-right motion - acceleration to the right is positive; acceleration to the left is negative.

2. Vertical motion - acceleration upward is positive; acceleration downward is negative. Note that the acceleration of gravity, $\overline{a}_g$, acts downward. Hence, $\overline{a}_g = -9.8\ m/s^2$.

In formula form, acceleration may be expressed as

$$\overline{a} = \frac{\overline{v}_f - \overline{v}_i}{t}$$

where $\overline{a}$ = acceleration
$\overline{v}_f$ = final velocity
$\overline{v}_i$ = initial velocity
t = time during which the velocity changed from $\overline{v}_i$ to $\overline{v}_f$.

Summary of Equations of Motion

We can rewrite the equations given earlier to include the quantities that involve direction.

1. $\overline{v}_{av} = \frac{\overline{d}}{t} = \frac{\overline{v}_f + \overline{v}_i}{2}$

2. $\overline{a} = \frac{\overline{v}_f - \overline{v}_i}{t}$ or $\overline{v}_f = \overline{v}_i + \overline{a}t$

3. $\overline{d} = \frac{\overline{v}_i + \overline{v}_f}{2}\, t$

4. $\overline{d} = \overline{v}_i t + \frac{1}{2}\overline{a}t^2$

5. $\overline{v}_f^{\,2} = \overline{v}_i^{\,2} + 2\,\overline{a}\,\overline{d}$

In the above equations, $\overline{a}$ is assumed to be uniform.

Study the following examples.

EXAMPLE 5

What is the final velocity of a body starting from rest and accelerating uniformly to the left at 15 m/s² for 5.0 s? What is the final speed?

Given: $\overline{v}_i = 0$

$\overline{a} = -15\ \text{m/s}^2$

$t = 5.0\ \text{s}$

To find: $\overline{v}_f$

Solution: $\overline{v}_f = \overline{v}_i + \overline{a}t$

$\overline{v}_f = 0 + (-15\ \text{m/s}^2)(5.0\ \text{s})$

$= -75\ \text{m/s}$

Speed $= v_f = 75$ m/s

The final velocity is 75 m/s to the left. Final speed is 75 m/s.

EXAMPLE 6

What was the initial velocity of a body that was accelerated to the right at 8.00 m/s^2 for 3.50 s to a final velocity of 32.0 m/s to the right?

Given:

$\bar{a} = +8.00\ m/s^2$

$t = 3.50\ s$

$\bar{v}_f = +32.0\ m/s$

To find: $\bar{v}_i$

Solution:

$\bar{v}_f = \bar{v}_i + \bar{a}t$

$\bar{v}_i = \bar{v}_f - \bar{a}t$

$= 32.0\ m/s - (8.00\ m/s^2)(3.50\ s)$

$= 32.0\ m/s - 28.0\ m/s$

$= +4.0\ m/s$

The initial velocity was 4.0 m/s to the right.

EXAMPLE 7

For the body described in **EXAMPLE 6**, what would have been the displacement of the body after the 3.50 s of acceleration at 8.00 m/s^2?

Given:

$\bar{v}_i = +4.0\ m/s$

$t = 3.50\ s$

$\bar{a} = +8.00\ m/s^2$

To find: $\bar{d}$

Solution:

$\bar{d} = \bar{v}_i t + \frac{1}{2}\bar{a}t^2$

$\bar{d} = (+4.0\ m/s)(3.50\ s) + \frac{1}{2}(+8.00\ m/s^2)(3.50\ s)^2$

$= +14\ m + 49\ m$

$= 63\ m$

The displacement would have been 63 m to the right.

EXAMPLE 8

A stone is thrown vertically upward with an initial speed of 30 m/s. How high will it go? What is its displacement at the top of its flight?

Given: $\bar{v}_i = 30$ m/s
$\bar{a}_g = -9.8\ \text{m/s}^2$
$\bar{v}_f = 0$ (The stone will keep going up until its speed is zero.)

To find: $\bar{d}$

Solution: Often you can save some calculations by selecting the more suitable formula. You can see the two solutions here. The alternate solution saves some steps though both solutions are equally correct.

$$\bar{v}_f = \bar{v}_i + \bar{a}_g t$$

$$t = \frac{\bar{v}_f - \bar{v}_i}{\bar{a}_g}$$

$$= \frac{0 - (+30\ \text{m/s})}{-9.8\ \text{m/s}^2}$$

$$= 3.06\ \text{s} = 3.1\ \text{s}$$

$$\bar{d} = \bar{v}_i t + \tfrac{1}{2}\bar{a}t^2$$

$$= (30\ \text{m/s} \times 3.06\ \text{s}) + \tfrac{1}{2}(-9.8\ \text{m/s}^2)(3.06\ \text{s})^2$$

$$= +91.8\ \text{m} - 45.88\ \text{m}$$

$$= 45.9\ \text{m} = 46\ \text{m}$$

The height will be 46 m. The displacement at the top will be +46 m.

Alternate Solution

$$\bar{v}_f^2 = \bar{v}_i^2 + 2\bar{a}_g\bar{d}$$

$$\bar{d} = \frac{\bar{v}_f^2 - \bar{v}_i^2}{2\bar{a}_g}$$

$$= \frac{0 - (30\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)}$$

$$= \frac{900}{19.6}\ \text{m}$$

$$= 45.9\ \text{m} = 46\ \text{m}$$

Do the following problems.

PROBLEMS

1. A car starts up from rest, accelerates uniformly for 1.0 min and acquires a velocity of 30 km/h to the right. What will be the displacement of the car in this time interval? What is the car's acceleration during this period?

2. A body accelerates from rest for 15.0 s to a velocity of 60 m/s to the right. It remains at that velocity for 4.0 s, and then accelerates for 12.0 s to a velocity of 24 m/s to the left. Find the following things.

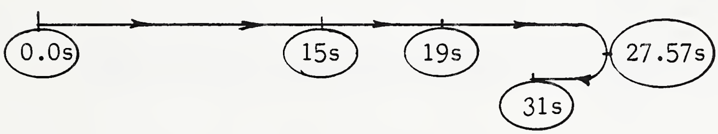
   (a) The acceleration during the first 15.0 s period.

   (b) The acceleration during the next 4.0 s period.

(c) The acceleration during the final 12.0 s period.

(d) The displacement of the body from its starting point to its position at the end of the 12.0 s period.

(e) The total distance travelled by the body. Notice that the direction of motion changes in the last 12 s. So the total distance travelled should be found to be larger than the displacement found in (d). The motion can be represented by the following diagram:



Lesson Summary

This lesson covered the following major topics.

1. Motion along a straight line, or one-dimensional motion, can be defined partially in terms of the ideas of position, distance, speed and rate of change of speed.

2. Certain equations of motion involving distance, speed, and rate of change of speed can be used to do calculations that provide a way of partially describing motion.

3. Description of motion - a body may be described as being in motion with respect to an observer if a line joining the object to the observer changes in either length or direction.

   Motion must be considered as being definable only with respect to some specified point.

4. By using plus and minus signs for direction according to an accepted convention, the quantities displacement, velocity, and acceleration can be used in the equations of motion.

5. The use of the following equations of motion was outlined.

$$\bar{v}_{av} = \frac{\bar{d}}{t} = \frac{\bar{v}_i + \bar{v}_f}{2}$$

$$\bar{v}_f = \bar{v}_i + \bar{a}t$$

$$\bar{d} = \frac{\bar{v}_i + \bar{v}_f}{2}\, t$$

$$\bar{d} = \bar{v}_i t + \tfrac{1}{2}\bar{a}t^2$$

$$\bar{v}_f^2 = \bar{v}_i^2 + 2\bar{a}\bar{d}$$

End of Lesson B

PHYSICS UPGRADING
LESSON C

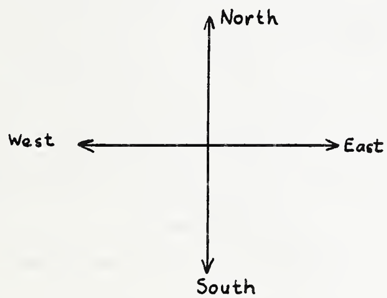
DO NOT send this lesson for correction

SCALAR AND VECTOR QUANTITIES

In the previous lesson when we dealt with quantities that involved direction, we were dealing with **vector** quantities. Vector quantities are those which have magnitude and direction. Quantities which have magnitude but not direction are **scalar** quantities. Examples of vector quantities are displacement, velocity and acceleration. Examples of scalar quantities are distance, speed, and rate of change of speed.

With one dimensional motion we dealt fairly easily with vector quantities by using a convention that involved positive and negative signs: motions to the right were considered positive and motions to the left were considered negative. In this lesson we will be considering two dimensional motion, so it will not be so easy to represent directions. Two dimensional motion involves motion in a plane, or along a flat surface. For many purposes, we tend to consider the earth's surface to be flat. Using the earth's surface as a reference, we can define directions in terms of the points of the compass (north, south, east, west). Fig. A shows the directions cross which shall be our primary reference for defining directions in this lesson. Note that this direction convention is the same as that used on most maps.

Fig. A



In order to represent vector quantities in two dimensions, we shall draw lines with arrowheads on them. The lengths of the lines will represent the magnitudes of the vector quantities, and the arrowheads will show their directions. The details of how this is to be done will be given later.

Do the following exercise.

Exercise 1

1. Explain the difference between vector and scalar quantities.

____

____

____

____

2. For each of the following quantities, indicate in the blank provided whether it is a vector or a scalar quantity.

(a) 90 km north ______________

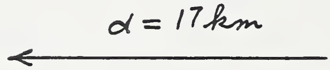
(b) 15 cm ______________

(c) the speed of light ______________

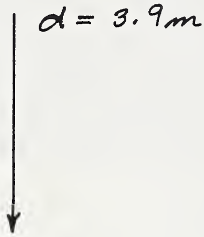
(d) the earth's force on the moon ______________

(e) the surface area of the moon ______________

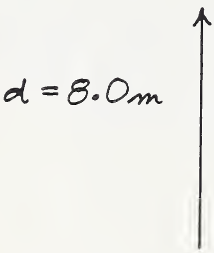
3. In the following diagrams a number of displacements are represented by lines with arrowheads on them. The arrowheads represent the direction of motion. Beside the lines appear distances. On the blanks provided, write the magnitudes and directions of the displacements. Refer to directions cross to find the directions.



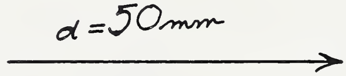
$\vec{d}$ = ____________



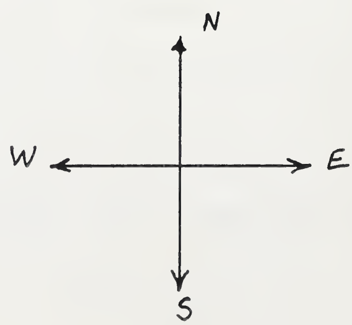
$\vec{d}$ = ____________



$\vec{d}$ = ____________



$\vec{d}$ = ____________



Representing Vector Quantities

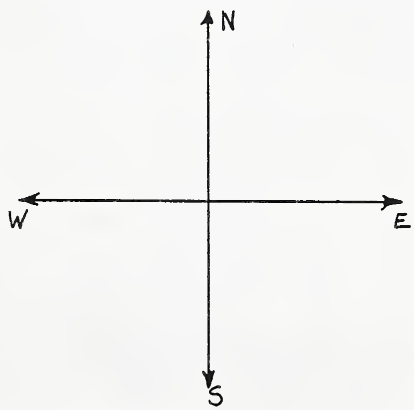
Any vector quantity such as displacement or velocity may be represented by a **line** (or a line segment). The length of this line indicates a **magnitude,** and an arrowhead placed at one end of the line represents the **direction** of the vector quantity.

One way of drawing a line to represent a vector quantity such as displacement would be to draw a line equal in length to the magnitude of the displacement in the same direction as the displacement. However, once we started to deal with displacements having magnitudes more than 20 cm, we would have difficulty in drawing lines to represent them because of lack of drawing room. It is more practical to draw shorter lines which are proportional in length to the magnitudes — in other words, to draw to scale.

The first step in making a scale drawing is to select the scale, and write it down in some form, such as **1 cm = 1 km** or **1 cm = 1 m.** In these examples, the equality sign (=) means "represents." Hence, **1 cm = 1 km** may be read as **1 cm represents 1 km.** The scale you select should be the most convenient one, both in terms of the amount of space you have for drawing, and in terms of the ease with which you can draw to that scale. Concerning this latter point, it is often more convenient to use units in whole numbers when scaling than to use fractions of units.

Once the scale has been selected, use the directions cross to determine how the directions are to be shown on the sheet on which you are drawing. Normally, to the top of the sheet is northward, to the bottom is southward, to the right is eastward, and to the left is westward.

Fig. B

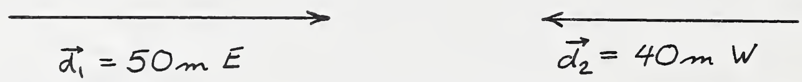


Use of the reference directions allows you to determine how the vector line should be oriented on your sheet. The line may now be drawn using a ruler and a protractor. It can be labelled with a letter with a small arrow above it. This way of symbolizing vector quantities allows the person writing to show whether a scalar or vector quantity is meant. Refer to the following sample vector representations. (Note that instead of drawing small arrows, you can draw small bars above the letters.)

Displacements

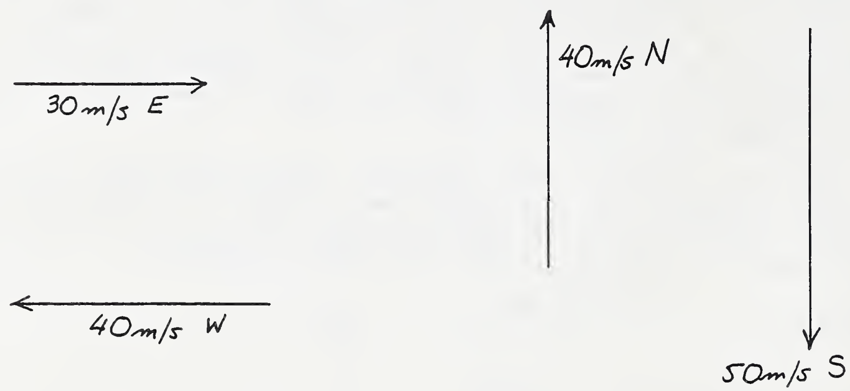
Fig. C

Scale: 1 cm = 10 m



Velocities

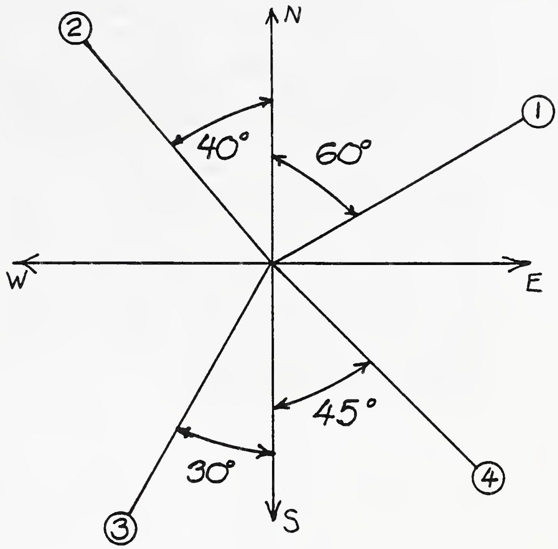
Scale: 1 cm = 10 m/s



More on Direction

So far in this lesson, the directions of the vector lines have been parallel to the north-south and east-west lines of the directions cross. However, as you probably know, vector quantities might have directions that are not parallel to those lines. In order to indicate such directions, it is necessary to make measurements of angles. To do this, a protractor should be used. (If you do not have a protractor now, you should obtain one before you continue with this lesson.)

Fig. D



As with making measurements of length, to measure angles it is useful to have a convenient starting point or reference point. Actually, to make measurements of angles on a surface, not only a reference **point** is needed, but also a reference **line** is needed from which to make measurements. In Fig. D, several measurements of angles have been made. Both a reference point and a reference line are involved in those measurements. The reference point is the point at which the N-S and E-W lines of the directions cross intersect or cross each other. The reference line is the N-S line from which the angles are indicated. Note that each of the lines drawn at an angle (other than 90°) to the N-S line is numbered ①, ②, ③, ④. In order to give the directions of the numbered lines, both the angle and its direction of measurement from the N-S line must be specified, as indicated below.

①	60°	east of north
②	40°	west of north
③	30°	west of south
④	45°	east of south

Note the following things.

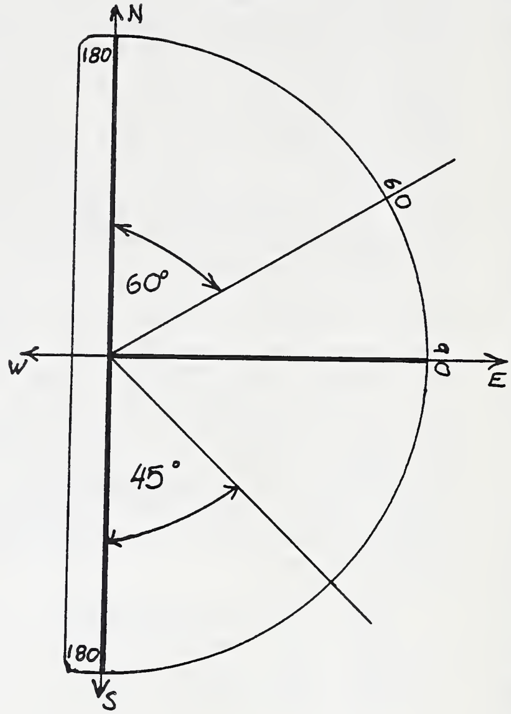
1. A line sloping above the E-W line, and to the right of the N-S line is **east of north.**
2. A line sloping above the E-W line and to the left of the N-S line is **west of north.**
3. A line sloping below the E-W line and to the left of the N-S line is **west of south.**
4. A line sloping below the E-W line and to the right of the N-S line is **east of south.**

Thus, for the lines drawn on a piece of paper, upward sloping lines are associated with north, and downward sloping lines are associated with south. Also, left and right are associated with west and east, respectively.

If you are not clear on how to use a protractor to make the measurements of angles such as those in Fig. D, study the following notes and figures before you go on to do the next set of exercises.

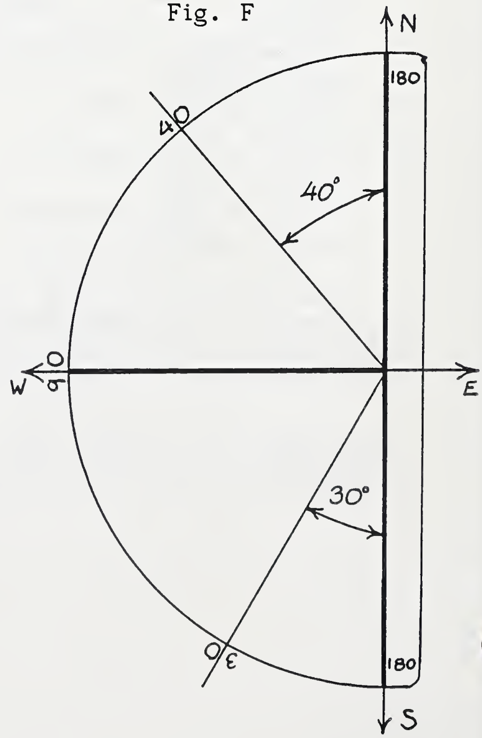
A protractor is illustrated in Fig. E. It is positioned so that the 180° line lies over the N-S line, and the 90° line lies over the E-W line, with the 90° line toward the right side of the page. When this is done, the directions of 60° east of north and 45° east of south can be easily read from the protractor. Take your protractor and place it in the same position as shown in Fig. E, and you will be able to see how the angles can be measured. Notice that the direction of a line toward the east is either 90° east of north, or 90° east of south, but that the direction should be specified simply as being **east.**

Fig. E



In Fig. F, a protractor is illustrated positioned so that it can measure angles west of the N-S line. The 180° line of the protractor lies over the N-S line and the 90° line lies over the E-W line and pointed to the left. If you position your protractor in a similar way, you will be able to see how the angles of 40° west of north and 30° west of south can be measured. Also, notice that the direction of a line toward the west is either 90° west of north or 90° west of south, but that the direction should be specified as being **west.**

Fig. F



Exercise 2

Use a protractor and ruler to complete these exercises.

1. Represent a displacement of 100 m north by a vector. (Use the scale given. Indicate the direction of the displacement by an arrowhead. Label the line representing this vector quantity $\bar{d}$ = 100 m north).

   Scale: **1 cm = 20 m**

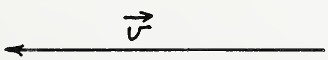
2. What is the displacement (magnitude and direction) labelled $\bar{d}$ in the following diagram?

   Scale: **1 cm = 10 m**

$\bar{d}$ = ______________________________
(Fill in the blank)

3. What velocity is represented by $\bar{v}$ in the following diagram?

   Scale: **1 cm = 10 m/s**



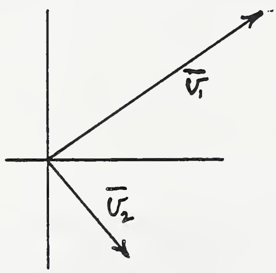
$\bar{v}$ = ______________________________

4. Represent a velocity of 70 m/s south by a vector. Label it $\overline{v}$.

Scale ____________ = ____________

5. Give the magnitudes and directions of the vector quantities represented in the following diagrams:

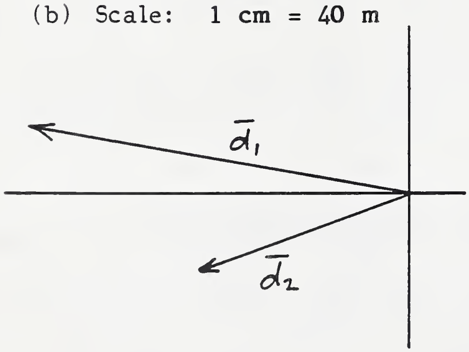
(a) Scale: 1 cm = 25 m/s



$\overline{v}_1$ = ____________________

$\overline{v}_2$ = ____________________

(b) Scale: 1 cm = 40 m



$\overline{d}_1$ = ____________________

$\overline{d}_2$ = ____________________

6. Draw lines representing the following vector quantities:

(a) 250 m 75° east of north.
Scale: 1 cm = 50 m

(b) 8.0 m/s 65° west of north
Scale: 1 cm = 1.0 m/s

(c) 750 m/s 80° west of south
Scale: 1 cm = 75 m/s

(d) 20 m 15° east of south
Scale: 1 cm = 5.0 m

Vector Diagrams 1

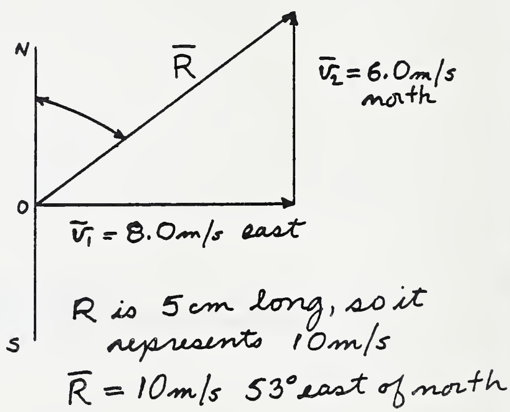
Vectors can be used to solve certain kinds of problems. One way of doing this is to use vector diagrams in which vector lines are drawn using the same scale and correct directions, and are combined to obtain a **resultant.** In physics, the term **resultant** often is used to name the vector quantity that results when vectors are combined.

The simplest vector diagram involves two vector lines which are joined together, with a resultant vector line giving the result of the combination. Fig. G shows how a vector diagram would be used to solve the following problem.

What is the resultant velocity of 8.0 m/s east, and 6.0 m/s north? To solve this problem, a suitable scale is selected first. Then, a line representing 8.0 m/s east is drawn. From the arrowhead end of this line, the line representing 6.0 m/s north is drawn. To draw the resultant, **start at the beginning of the first vector line (point 0) and draw a line to the arrowhead of the second vector line.** Measurements of the length of the resultant line and the angle between it and the N-S line at 0 give the magnitude and direction of the resultant respectively. This procedure is summarized below.

Fig. G

Scale: 1cm = 2.0m/s



1. Select an appropriate scale.

2. Draw the first vector line to scale and in the correct direction from the starting point, 0.

3. Draw the second vector line to the same scale and in the correct direction, starting from the tip of the arrowhead of the first vector line.

4. Draw the resultant vector line from the starting point, 0, to the tip of the arrowhead of the second vector line.

5. Measure the length of the resultant line to find the magnitude of the resultant, and the angle between it and the N-S line at 0 to determine the direction of the resultant.

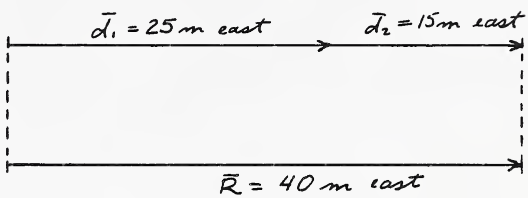
Note that an arrowhead should be included at the end of each vector line in order to clearly show the direction of the vector line.

There are two cases in which the procedure described above cannot be followed exactly. These two cases are: (1) when the two vectors are in the same direction; and (2) when the two vectors are in opposite directions. However, it is possible to draw vector diagrams in these two cases, as illustrated in the following notes.

1. What is the resultant of displacements of 25 m east and 15 m east?

Fig. H

Scale: 1cm = 5m



The method of solving this using a vector diagram is shown in Fig. H. However, it is simpler to not bother with drawing a vector diagram. Since the vector quantities are in the same direction, it is easier to add them:

$$\bar{R} = \bar{d}_1 + \bar{d}_2 = 25 \text{ m east} + 15 \text{ m east}$$
$$= 40 \text{ m east}$$

2. What is the resultant of displacements of 25 m east and 15 m west?

Fig. I

Scale: 1cm = 5m

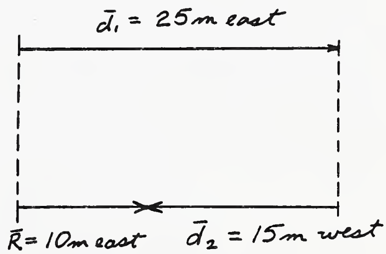


Fig. I shows the method of solving this problem using a vector diagram. However, often it is simpler not to draw the vector diagram. Since the vector quantities are in opposite directions, it is easier to assign a minus sign to one direction (west), and a plus sign to the opposite direction (east):

$$\bar{R} = \bar{d}_1 + \bar{d}_2 = 25 \text{ m east} + 15 \text{ m west}$$
$$= +25\text{m} + (-15 \text{ m})$$
$$= +10\text{m}$$
$$= 10 \text{ m east.}$$

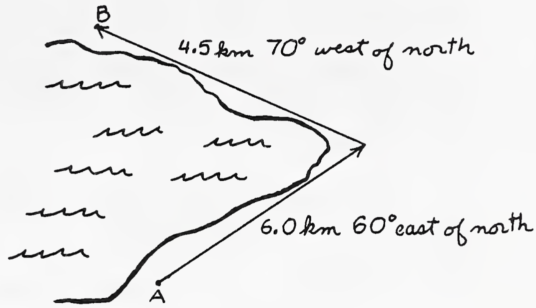
Complete **Exercise 3** using vector diagrams.

Exercise 3

1. A car travels 600 km west and then 450 km south. Find the resultant displacement.

2. An aircraft with a velocity of 400 km/h east meets a wind of 180 km/h north. Find the resultant velocity of the aircraft. (Assume that the effect of the wind's velocity would be fully transferred to the aircraft.)

3. A fisherman is near one end of a lake at point A, and he wishes to get from A to point B on the other side. See the diagram. What would be his displacement if he travelled from A to B? Note that the lines drawn in the diagram are not to any particular scale.



Vector Diagrams 2

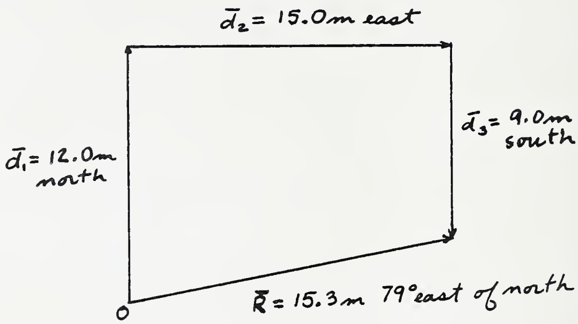
In the preceding section, we examined the method of drawing vector diagrams when two vector quantities are involved initially, and a resultant is to be found. Often, this method is good enough to solve problems involving vector quantities. However, sometimes more than two vector quantities are involved initially, and it is necessary to find a resultant. To do this using a vector diagram we can use basically the same procedure as before, as illustrated in Fig. J.

The problem is: What is the resultant of displacements of 12.0 m north, 15.0 m east, and 9.0 m south?

As before, a scale must be selected, and the vector lines are drawn according to the scale and in the directions specified. Note that $\overline{d}_1$ begins at 0, $\overline{d}_2$ begins at the tip of the arrowhead of $\overline{d}_1$, and $\overline{d}_3$ begins at the tip of the arrowhead of $\overline{d}_2$. The resultant is drawn from 0 to the tip of the arrowhead of $\overline{d}_3$. The only difference between this method and the one given earlier is that $\overline{d}_3$ is included, and the resultant must be drawn to the tip of the arrowhead of $\overline{d}_3$. This method can be used with any number of vector lines, from two up. Fig. K and L show the use of this method to solve multivector problems.

Fig. J

Scale: 1.0cm = 3.0m



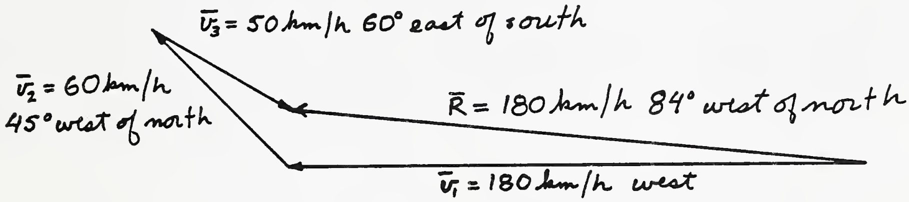
EXAMPLE

A search and rescue aircraft is flying low through mountain valleys and passes. At one point in a gap between several valleys, it has a velocity of 180 km/h west and is acted upon by two winds blowing down two different valleys. One wind has a velocity of 60 km/h 45° west of north, and the other has a velocity of 50 km/h 60° east of south. What is the resultant of these velocities?

The method of doing this is shown in Fig. K.

Fig. K

Scale: 1.0 cm = 20 km/h



Resultant velocity is 180 km/h 84° west of north.

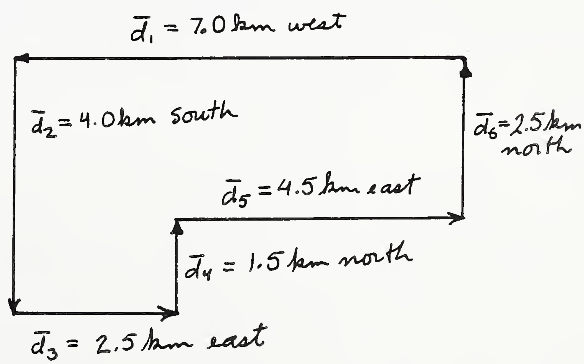
EXAMPLE

Find the resultant displacement of the following displacements: 7.0 km west, 4.0 km south, 2.5 km east, 1.5 km north, 4.5 km east, 2.5 km north. The method of doing this is shown in Fig. L.

Note that six vector lines are involved in the diagram shown in Fig. L. Also, note that when all six of the lines have been drawn, the last vector line ends at the original starting point. This means that no resultant line needs to be drawn, or that the resultant is equal to zero.

Fig. L

Scale: 1.0 cm = 1.0 km



Since no resultant vector line needs to be drawn, the resultant displacement is zero.

Do the following exercise.

Exercise 4

1. Find the resultant of 30 km/h north, 40 km/h 45° west of north and 70 km/h 65° east of south.

2. A man walks from point 0 as follows: 10.0 km 15° west of north, then 20.0 km 75° west of south, then 15.0 km east, and then 5.0 km south to reach point A. Find the following things.

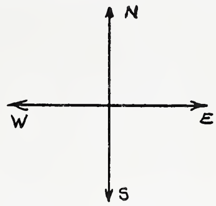
   (a) His resultant displacement with respect to 0.

   (b) His displacement from A if he wished to have a resultant displacement of zero with respect to point 0.

Lesson Summary

This lesson covered the following major topics.

1. Vector quantities may be represented by lines drawn to scale in the appropriate directions, according to an accepted convention.



2. Directions may be conveniently specified using angles measured from the N-S line using a protractor.

3. To find the resultant of two or more vector lines, the vector lines should be drawn with the first vector line starting at a point, such as 0, and the next vector line beginning at the tip of the arrowhead of the first vector line. Then, other vector lines may be drawn from the tips of the arrowheads of preceding vector lines. The resultant should be drawn from the starting point, 0, to the tip of the arrowhead of the last given vector line.

End of Lesson C

PHYSICS UPGRADING
LESSON D

DO NOT send this lesson for correction

FORCE, INERTIA, AND MASS

The Explanation of Motion

In our study of motion so far, we have been concerned with kinematics, which deals with the description of motion. To explain why motion occurs, we must study the area of physics known as **dynamics.** This approach to the study of motion, starting with a description of what is happening, and then trying to explain why it has happened, is a useful approach in other areas besides the physics of motion. Usually, it is necessary to know what really occurred before trying to explain it, especially if you are trying to solve a problem.

A useful explanation is one that not only is consistent with a particular law, but also one that can be used to predict what will happen in a particular situation. An explanation which is logically consistent with a law is not very useful if it does not provide a way of making predictions which can be checked or tested. Thus, for an explanation to be satisfactory scientifically, it must involve the following three features:

(1) It must be logically consistent with certain laws or rules.

(2) It must provide a way of making predictions.

(3) The predictions that are derived from it must be testable, and once tested must be accurate.

QUESTIONS

1. What is the difference between kinematics and dynamics?

____

____

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2. The following are concepts of kinematics or dynamics: speed, mass, force, distance, time, kinetic energy, position, and friction. Which four of these are concepts of kinematics?

(a) ____

(b) ____

(c) ____

(d) ____

3. (a) For a scientific explanation to be satisfactory, what features should it have?

____

____

____

(b) If you test a scientific explanation and find the predictions inaccurate, what should be done with this explanation?

____

____

____

____

Familiar Forces and Units of Force

An important concept in the area of dynamics is the idea of force. A force that most people are familiar with is the force of weight. We can feel our own weights, especially if we do something that requires that we lift ourselves — such as exercises like chin ups, push ups or sit ups. We can feel the weight of other things when we try to lift them. Another force that people experience is the force of friction. Friction forces act between two surfaces which are in contact with each other and they tend to act against the motion of one surface with respect to the other. If there is a large friction force, it may be very difficult to move something. For example, it might be much more difficult to slide a desk across a carpeted floor than across a smooth tiled floor because the friction force between the carpet and the desk is much larger. On the other hand, very small friction forces might cause difficulties also, as in the case of walking across smooth ice.

In this course, most of the forces will be expressed in one unit, called the **newton** (after Sir Isaac Newton). **N** is the symbol for the newton. We will give a more formal definition of the newton later, but to get a feeling for what a newton involves, consider the following:

The weight of two golf balls is about 1 N.

The weight of a flashlight battery (a D cell) is about 1 N.

The weight of a softball is about 2 N.

The weight of a two litre carton of milk is about 20 N.

Forces as Vector Quantities

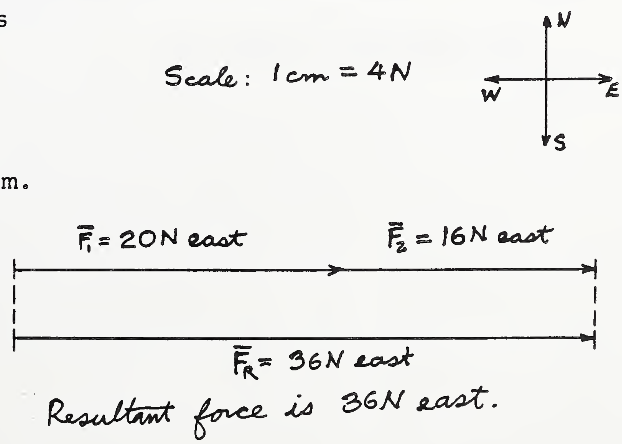
The idea of force as simply a push or a pull exerted on an object is an easy one to understand, since it is so closely related to everyday experience. In order to move around throughout each day, a person must exert forces on a number of different objects. For example, if a person walks out of a house, he/she must exert forces on the floor through legs and feet as well as exerting forces through arms and hands on the door in order to open and close it. It is clear that the direction as well as the size of each force exerted is important. Inside a house, if you turn the doorknob and **push** on an inward-opening door, you won't open it; such a door must be **pulled** in order to be opened. This illustrates the necessity of considering forces as **vector** quantities.

We can deal with forces as vector quantities by using the same kinds of methods as we used earlier; that is, we can use lines with arrowheads to represent forces, and we can use vector diagrams to solve certain kinds of problems involving forces. Since the preceding lesson was concerned with representing vector quantities and drawing vector lines, you should be familiar with the methods of doing this. However, if you are not clear about some of the methods used, you should review the preceding lesson before continuing with this lesson.

The simplest situation in which two forces are involved (when it is necessary to find a resultant) occurs when the two forces act along the same straight line in the same direction. Such a problem can be solved using a vector diagram, but usually it is easier to simply add the sizes of the two forces to obtain the size of the resultant, and indicate its direction as being the same as the direction of the original forces.

Example 1

Fig. A



What is the resultant of forces of 20 N east and 16 N east?

A vector diagram solving this problem is shown in Fig. A. Below, the problem is solved without the use of a vector diagram.

$\overline{F}_R = \overline{F}_1 + \overline{F}_2$

$\overline{F}_R$ = 20 N east + 16 N east

$\overline{F}_R$ = 36 N east

Another simple situation involves two forces which act along the same straight line, but in opposite directions. As before, a vector diagram may be used to find the resultant, but it may be easier to make the force in one direction positive, and the force in the opposite direction negative. Then, they can be combined to find the resultant.

Example 2

What is the resultant of forces of 20 N east, and 16 N west?

A vector diagram solving this problem is shown in Fig. B. Below, the problem is solved without the use of a vector diagram.

Make east positive, and west negative.

$\overline{F}_R = \overline{F}_1 + \overline{F}_2$

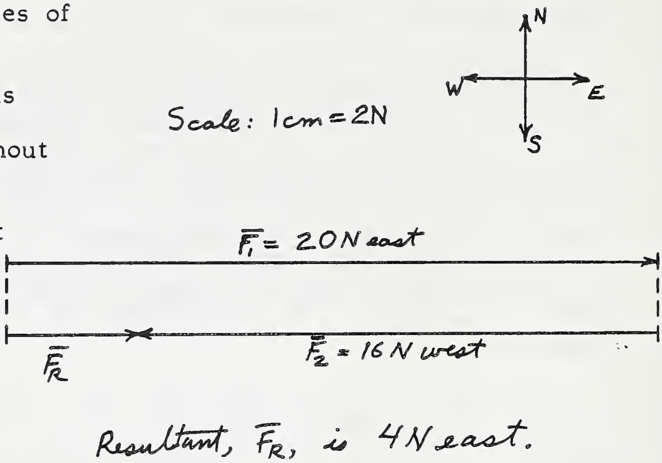
$\overline{F}_R = 20 \text{ N east} + 16 \text{ N west}$

$\overline{F}_R = +20 \text{ N} + (-16 \text{ N})$

$\overline{F}_R = +4 = 4 \text{ N east}$

Resultant force is 4 N east.

Fig. B



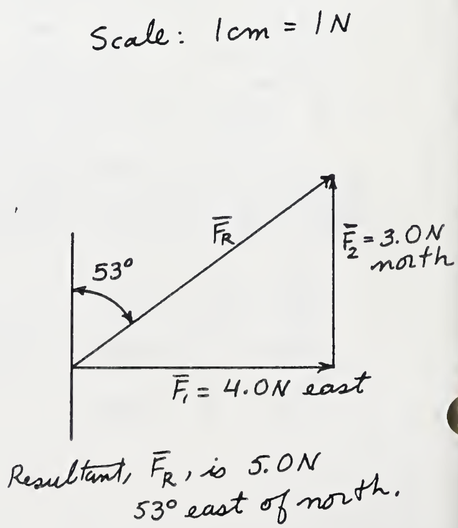
Of course, forces do not always act along the same straight line, and to find the resultant in such cases it is useful to draw vector diagrams according to the rules given in the preceding lesson. The following examples show the application of those rules to problems involving forces.

Example 3

Find the resultant of two forces, one 4.0 N east, and the other 3.0 N north.

The solution is shown in Fig. C.

Fig. C

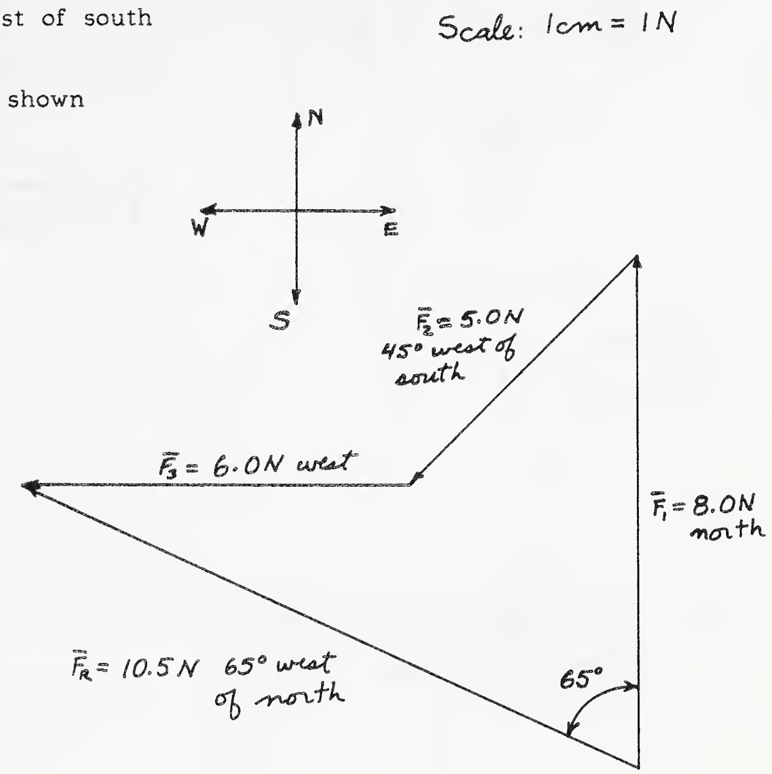


Example 4

Find the resultant of the following set of forces: 8.0 N north
5.0 N 45° west of south
6.0 N west

The vector diagram is shown in Fig. D.

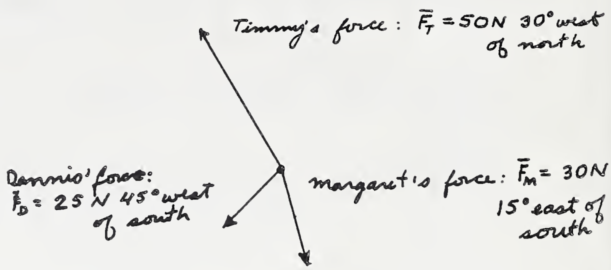
Fig. D



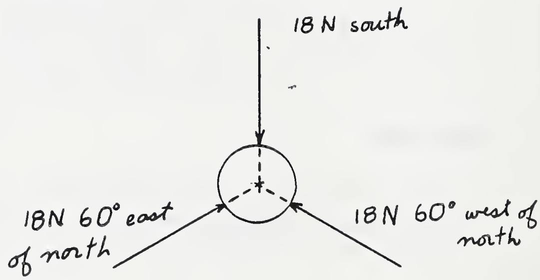
PROBLEMS

1. Two children have a dispute over a toy. Dennis pulls with a force of 25 N, and Margaret pulls in the opposite direction with a force of 30 N. What is the resultant force on the toy?

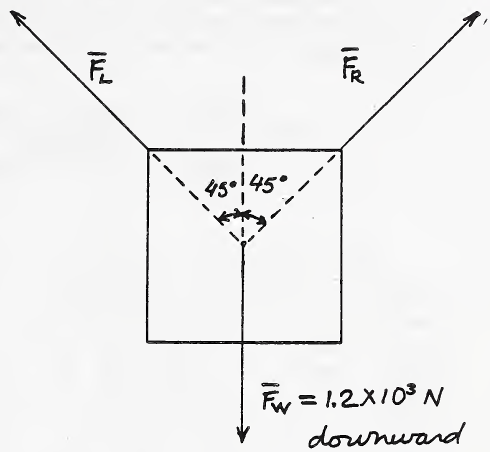
2. While Dennis and Margaret are struggling, a larger child, Timmy, arrives on the scene and decides to join in. This changes the arrangement of the forces applied to the toy, as shown in the diagram. Using a vector diagram, find the resultant force on the toy.



3. Three forces act on a spherical object as illustrated in the following diagram. Find the resultant of these forces using a vector diagram.



4. A square sheet of metal weighing 1.2×10^3 N is to be supported by two cables, as shown in the diagram. The forces in the two cables, F_L and F_R, are to be equal in size. Using a vector diagram, find the size of each of the forces in the cable.



Newton's First Law of Motion

Although Newton was the first to formally enunciate the statement that is known as Newton's first law of motion, Galileo provided the line of reasoning that led to it. Newton's first law of motion is stated as follows:

> **Every object continues in a state of rest or of uniform motion in a straight line unless acted upon by an external unbalanced force.**

This statement tells us that every body resists any changes in its speed or direction of motion. If at rest the body will remain at rest. If moving at a constant velocity, it will continue to do so in the absence of any external unbalanced forces. Note that the reference is to an **unbalanced** force. What is meant by the term "unbalanced force?" We can answer this question by recalling that when we drew vector diagrams, we sometimes obtained resultants of zero. When the vectors involved in the diagrams were forces, this meant that when the forces were combined, they all balanced each other. Thus, for balanced forces, the resultant is zero. For unbalanced forces, the resultant force is not equal to zero. Often, the term "unbalanced force" refers specifically to the non-zero resultant of several forces acting on a body.

From this law, a definition of inertia can be derived. One form of the definition of inertia is that inertia is the tendency of a body to remain at rest or to continue moving uniformly in a straight line.

Many common occurrences illustrate the inertia of bodies. In a train, you appear to be thrown backward as the train begins to move forward. Your inertia tends to keep you at rest, while the train drives out from under you. When you shovel snow, you first get the shovel and the snow moving, then stop the shovel with your arms. The snow continues in motion, exhibiting its property of inertia. A motor boat drifting after the engine is stopped, a curling stone sliding down the ice, your tendency to fall forward when the bus you are riding is suddenly stopped, are all examples of inertia. Note that inertia is not a force, it merely describes a property of all objects. Sometimes the property that we have named inertia is also called mass.

QUESTIONS

1. What is inertia?

___

___

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2. First of all you are standing in a bus which starts to accelerate from rest - then you fall towards the back of the bus. After you have regained your composure the driver applies the brakes suddenly. Whereupon you fall towards the front of the bus.

   In which case (first, second) is inertia shown by a stationary body? ____________

   In which case is inertia shown by a moving object? ____________

3. (a) What is meant by the phrase "balanced forces?"

   ____

   ____

   ____

   (b) What is meant by the phrase "unbalanced forces?"

   ____

   ____

   ____

4. Two men must push a stalled car to the side of a road. They find that it is quite difficult to get the car to move at first, but that once it is moving it is much easier to push the car. Why is the car easier to push once it is in motion?

   ____

   ____

   ____

   ____

5. From what you know about inertia, does the use of seat belts in moving vehicles make sense? Give an explanation for your answer.

____

____

____

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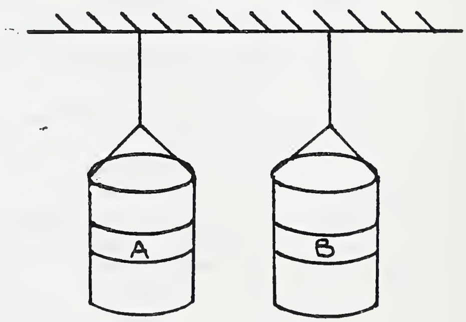
Inertia and Mass

When they are first met, the ideas of inertia and mass often do not seem to be very clear. Sometimes, this is especially true for the idea of **mass.** Mass was once considered to be a way of representing the amount of matter in a body. After all, it usually seems to be true that when a body appears to have more matter in it – that is, it looks bigger – it is more "massive." However, one problem with looking at mass in this way is that it may lead to some confusion. This is because what is being done is that volume is being used as a guide for estimating mass. **Volume** is a direct indication of the size of a body; that is, it is a way of measuring how much space a body takes up. However, estimating volume is not a reliable way of determining how much matter is in a body. To understand this, we can do a simple thought experiment.

Suppose that there are two identical cans suspended from the ceiling by strings. Refer to Fig. E. The tin cans have covers on their tops and are labelled with the letters A and B. One can is empty, and the other can is full of concrete. By just looking at the cans, can you tell which one is empty and which one is full of concrete? Unless you have X-ray vision, you probably will not know which is which. How can you find out?

Fig. E



Probably, the easiest way to do this is to give each can a light push with your hand. You will be able to feel the difference in the resistance to the force applied by your hand. If can B is much harder to start moving than can A, you will know that can B is full of the concrete while can A is the empty one.

This thought experiment illustrates some important things. It shows that a judgement about masses of bodies cannot be done reliably only on the basis of volume. Also, it shows that we can find out about the masses of bodies by applying forces to the bodies, and observing how these forces affect the motions of the bodies. In other words, it shows that inertia and mass are closely related.

The relationship of mass and inertia is given in the following statement:

mass is a measure of the inertia of a body.

Using the above statement and the method described in our thought experiment, we can provide a further definition. This definition is provided below.

> **It is possible to determine which of two masses is greater by applying equal forces to each mass for equal time periods and observing which mass speeds up less. That mass is the greater one.**

The above statement is a more detailed way of saying that mass is the measure of the inertia of a body. It is what is sometimes called an **operational definition.** An operational definition is a statement which describes an activity in order to explain the meaning of a word. Operational definitions are very important in physics and other sciences because they provide procedures which make clear how you can test for the presence of a phenomenon.

In making measurements of mass, the unit which is used often is the **kilogram.** Approximate masses of some objects with which you may be familiar are given below. Note that **kg** is the symbol for kilogram.

Mass of two golf balls is about 0.1 kg.

Mass of a flashlight cell (D cell) is about 0.1 kg.

Mass of a softball is about 0.2 kg.

Mass of a two litre carton of milk is about 2 kg.

Something that should be remembered when dealing with mass (or inertia) is that it is **not** the same as weight. Weight is a **force,** and is a vector quantity, having both magnitude and direction. Mass (or inertia) is **not** a force, and it is a **scalar** quantity, since it has magnitude, but not direction. Mass (or inertia) is the same regardless of the direction used in the measurement. In other words, the same tendency of a body to resist a change in its state of rest or of uniform motion will appear even though the direction of the unbalanced force changes. This difference between mass and force will be emphasized further in a later lesson when we learn more about the relationship between weight and mass.

QUESTIONS

1. What is an operational definition?

____

____

____

2. Give an operational definition of mass.

3. What is the difference between volume and mass?

4. Is it possible for a body to have a much larger volume than another, but have a smaller mass? Explain this, giving an example.

5. A cargo handler in a space station works under the conditions known as weightlessness. There are several identical cartons floating in front of him; one of the cartons is full of gold bars, and the others are empty. How would he find the carton with the gold bars without opening the cartons?

6. What is the difference between weight and mass?

Lesson Summary

This lesson covered the following major topics:

Explanation of motion - Motion is described in the study of kinematics, and is explained in the study of dynamics. A satisfactory scientific explanation must have several features.

Familiar forces and units of force - Two kinds of forces with which most people are familiar are the forces of weight and friction. The newton, symbolized by **N**, is the unit of force often used in physics.

Forces as vector quantities - Forces have magnitude and direction, and hence are vector quantities. They can be represented by vector lines, and vector diagrams may be used to solve problems which involve finding the resultant of a group of forces acting on a body.

Newton's first law of motion.

Inertia and Mass - Mass is a measure of the inertia of a body. Operational defintions help to explain the meaning of words. The kilogram, symbolized **kg**, is the unit of mass often used in physics. Mass (or inertia) is not the same as weight.

End of Lesson D

PHYSICS UPGRADING
LESSON E

DO NOT send this lesson for correction

NEWTON'S SECOND AND THIRD LAWS OF MOTION

Newton's Second Law of Motion

Newton's first law of motion specifies the conditions under which a body's velocity will remain constant: when the resultant force acting on the body is zero, there will be no change in velocity (zero acceleration). The question of what occurs when the resultant force is not zero is dealt with by Newton's second law of motion. When a non-zero resultant force acts, there is change in motion of the body. From your everyday experience, you can visualize that if you are pushing a body and the body accelerates with an acceleration **'a'**, then the greater the push the greater the acceleration will be. You can say that acceleration is **directly** proportional to the force applied, which is the push in this case. Mathematically we can write,

$$\bar{a} \propto \bar{F}$$

Similarly you can visualize that if you apply the same push or force to a smaller mass it will accelerate more than if you apply the same push or force to a larger mass. Thus you can say that acceleration is **inversely** proportional to the mass of the body. Mathematically, we can write,

$$a \propto \frac{1}{m}$$

Combining these two facts, you can write

$$\bar{a} \propto \frac{\bar{F}}{m}$$

or

$$\bar{a} = k\,\frac{\bar{F}}{m}$$

Choosing proper units, so that $k = 1$,

we have $\bar{a} = \dfrac{\bar{F}}{m}$

The "word form" of Newton's second law can be stated as follows: **the acceleration of a body is in the same direction as the unbalanced force acting on the body, and varies directly with the magnitude of the unbalanced force, and inversely with the mass of the body.**

Note that if the unbalanced force is zero, acceleration will be zero, and velocity will be constant.

Thus the so-called 'force-equation' is obtained as

$$\bar{F} = m \cdot \bar{a}$$

Force = mass × acceleration

The most common unit of force is the **newton.** The newton is defined as the force that, applied to a 1 kg mass, will give it an acceleration of 1 m/s^2 i.e.

$$1\ N = 1\ kg \times 1\ m/s^2$$

Note that force and acceleration are vector quantities, and that mass is a scalar quantity. When an unbalanced force (or non-zero resultant force) acts on a body, the direction of the acceleration is always the same as the direction of the unbalanced force (sometimes the unbalanced force also is called the net force). Sometimes the formula form of Newton's second law is given as if the acceleration and force were not vectors: **F = ma.** Such usage is acceptable as long as it is remembered that the net force and the acceleration have the same direction.

EXAMPLE 1

What will be the acceleration of a 1000 kg car if an unbalanced force of 800 N is applied?

Given: $F = 800\ \text{N}$

$m = 1000\ \text{kg}$

To find: a

Solution:

$$\begin{aligned} a &= F/m \\ &= \frac{800\ \text{N}}{1000\ \text{kg}} \\ &= \frac{800\ \text{kg}\cdot\text{m/s}^2}{1000\ \text{kg}} \\ &= 0.800\ \text{m/s}^2 \end{aligned}$$

EXAMPLE 2

A pony with a mass of 300 kg pulls a cart with a mass of 1000 kg. With what force will the pony have to pull on the cart if he is to accelerate at 2.0 m/s^2 ? Assume no friction in the wheels.

Note: The force on the cart accelerates the cart only, therefore only the mass of the cart should be considered.

Given: $m = 1000\ \text{kg}$

$a = 2.0\ \text{m/s}^2$

To find: F

Solution:

$$\begin{aligned} F &= m\cdot a \\ &= 1000\ \text{kg} \times 2.0\ \text{m/s}^2 \\ &= 2000\ \text{kg m/s}^2 \\ &= 2.0 \times 10^3\ \text{N} \end{aligned}$$

Note that in **EXAMPLES** 1 and 2 acceleration and force have been treated as if they were scalar quantities. Since the acceleration and the unbalanced force have the same directions, no reference to direction has been made. In the following example, direction takes a more prominent role.

EXAMPLE 3

Two forces act on a body of 25 kg at the same time: 40 N east and 30 N north. What is the acceleration of the body?

Given: $\overline{F_1}$ = 40 N east

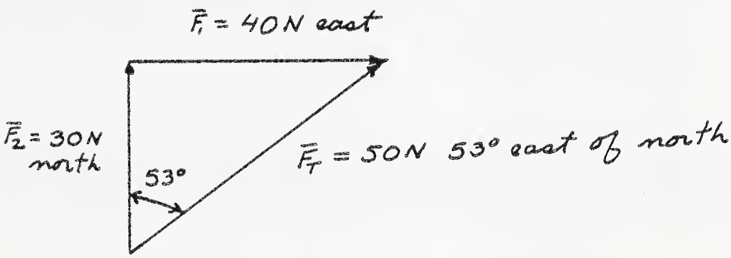
$\overline{F_2}$ = 30 N north

m = 25 kg

To find: $\vec{a}$

Solution: The resultant or unbalanced force must be found and then the acceleration can be found. A vector diagram is used to find the unbalanced (or net) force, F_T.

Scale: 1 cm = 10 N



By Newton's second law:

$$F_T = ma$$

$$a = \frac{F_T}{m}$$

$$= \frac{50\ \text{N}}{25\ \text{kg}}$$

$$= 2.0\ \text{m/s}^2$$

Since the unbalanced force and acceleration have the same direction, we have $\overline{a}$ = 2.0 m/s², 53° east of north.

QUESTIONS

1. State Newton's Second Law (a) in words, (b) in equation form.

   (a) ____

   ____

   ____

   (b) ____

2. (a) Can Newton's Second Law be applied in situations involving friction? ____________

   (b) If you apply a force of 10 N to an object and the force of friction is 4 N, what is the magnitude of the force that must be used in applying $\overline{F} = m\overline{a}$? ____________

3. The same unbalanced force is applied to bodies of masses 1 kg, 3 kg, and 5 kg. If the acceleration produced on the first body is 15 m/s^2, what will be the acceleration of the other bodies?

4. To push a book at constant velocity along a table requires a constant applied force. What is the value of the net force on the book?

   ____

5. A net force of 10 N gives an object a constant acceleration of 4.0 m/s^2. What is the mass of the object?

6. A mass of 15 kg on a frictionless surface is given an acceleration of $3.0\ m/s^2$ by an applied force. What was the value of the force?

PROBLEMS

Attempt the following problems and show your work.

1. A cart accelerates from rest to 6 m/s in 2 s. The force of friction is 5 N. If the cart is being pushed by an 11 N force, what is the mass of the cart?

2. A 1500 kg car moving at 15 m/s crashes into a wall and comes to stop in 0.50 s. Assume that the acceleration of the car while stopping was uniform. Calculate the collision force in the crash.

   Remember $\bar{a} = \frac{\Delta \bar{v}}{\Delta t}$.

3. A freight engine of mass 20 000 kg accelerates uniformly from rest to a velocity of 2.0 m/s in 5.0 s. If it is pulling a train of 20 cars each with a mass of 10 000 kg, what is the force in the coupling between the engine and the first car?

   (Hint: Is the mass of the engine needed?)

4. A 60 kg body has three forces acting on it: 18.0 N north, 10.0 N west, and 15.0 N 45° west of south. Find the acceleration (magnitude and direction) of the body.

Newton's Third Law

The third major contribution Newton made to our understanding of dynamics does not concern motion directly, but points out a different consequence of forces.

Strike your hand against a wall. Your hand will experience force on it. When you hit a ball with a bat, the bat exerts a force on the ball. At the same time, the ball exerts a force on the bat. These two forces are equal in magnitude and opposite in direction.

This is an example of Newton's Third Law of Motion which states that **if one object applies a force to another, the second object applies an equal and opposite force to the first object.**

Newton's Third Law clearly shows that:

1. forces always occur in pairs
2. each force of the pair acts on a different object
3. each force of the pair is equal in magnitude but acts in the opposite direction to the other.

We often refer to one force in the pair as an action force and the other as a reaction force. This provides us with another definition of Newton's Third Law as **"To every action there is an equal and opposite reaction."**

Sometimes, Newton's third law and its use produces some confusion. This may occur because the action and reaction forces referred to are mistakenly considered to be the same as two balanced forces. As you know, if two equal and opposite forces act on a body, the resultant force will be zero, and no acceleration of the body will occur. If the action-reaction couple is considered to be the same as two equal and opposite forces acting on the same body, then it may seem that Newton's third law is saying that **resultant forces acting on bodies always must be zero.** This is mistaken because the action-reaction couple **does not act on the same body.** Therefore, the action and reaction forces are not the same as equal and opposite forces that produce a resultant of zero. An action-reaction couple consists of two equal but opposite forces that occur on two bodies interacting with each other.

QUESTIONS

1. State Newton's Third Law of Motion.

____

____

2. Name the action and reaction forces in each of the following situations. Specify the objects on which the two action and reaction forces act.

(a) A man holds up a book with his hand.

action force _______________________________

reaction force _______________________________

(b) The earth exerts a gravitational force on the moon.

action force *the earth pulls on the moon*

reaction force *the moon pulls on the earth*

(c) The north pole of a magnet exerts an attractive force on the south pole of another magnet.

action force _______________________________

reaction force _______________________________

(d) A leaf falling to the ground pushes down on molecules of air.

action force _______________________________

reaction force _______________________________

Note that the two forces listed in each case make up an action-reaction couple.

3. A man is stranded near the centre of a frozen pond. Imagine that the pond surface is perfectly frictionless.

(a) Could he walk to shore?
(b) How would you suggest he could get to shore?

(a) __________

(b) ____

____

____

____

Weights and Mass

You have already learned that the inertia of an object provides a way of defining mass. The greater the mass of an object, the greater is its tendency to keep at a constant velocity. Therefore mass is often defined as the quantitative measure of the inertia of an object.

Weight is the resultant of all the gravitational forces on an object. Weight is not a good physical standard; it varies from place to place. Mass on the other hand is constant, at ordinary velocities, even when moved from place to place.

When a mass, m, is allowed to fall freely, it is the constant downward force of gravity on the mass that gives rise to its constant acceleration. If Newton's Second Law is applied to this motion, the force, F, is none other than the weight, F_W, of the body, and the acceleration, a, is the acceleration due to gravity, a_g. Therefore for falling bodies the force equation, $F = ma$, can be written as $F_W = ma_g$

or weight = mass × acceleration due to gravity,

Weight and force have both magnitude and direction and are therefore vector quantities. Mass, on the other hand, is a scalar quantity, because it has only magnitude. The distinction between weight and mass is illustrated by imagining a given body to be carried out into free space far removed from other bodies and their gravitational attraction. There, a body at rest will still have its mass, but its weight will be zero. Weight on the earth is due to the gravitational attraction of the earth and value of acceleration due to gravity is taken to be $9.81\ m/s^2$ or $9.8\ m/s^2$.

One often hears of a weight referred to in grams or kilograms. This is confusing because grams and kilograms are mass units, not weight units. This expression of weight in mass units should be avoided.

EXAMPLE 3

Calculate the weight of a body of mass 1.0 kg.

Given: $m = 1.0\ kg$

$a_g = 9.8\ m/s^2$

To find: F_W

Solution: $F_W = ma_g$

$= 1.0\ kg \times 9.8\ m/s^2$

$= 9.8\ N$

PROBLEMS

1. Calculate the weights of the following masses.

   (a) 1.50×10^2 kg

   (b) 16.0 kg

2. What is the mass of a body which has a weight of 1.87×10^3 N at the earth's surface?

Newton's Law of Gravitation

Nearly everyone has heard the story of how Newton, while sitting under an apple tree one day, was struck on the head by a falling apple. This incident set Newton to thinking and led to the discovery of Newton's universal law of gravitation, which states that, **"any two bodies attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them."** Written in symbols,

$$f \propto \frac{m_1 m_2}{d^2}$$

where m_1 and m_2 are two masses at a distance, d, apart, measured centre to centre. The above proportionality is changed into an equality by introducing a constant. Then the equation becomes:

$$F = G \frac{m_1 m_2}{d^2}$$

where $$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

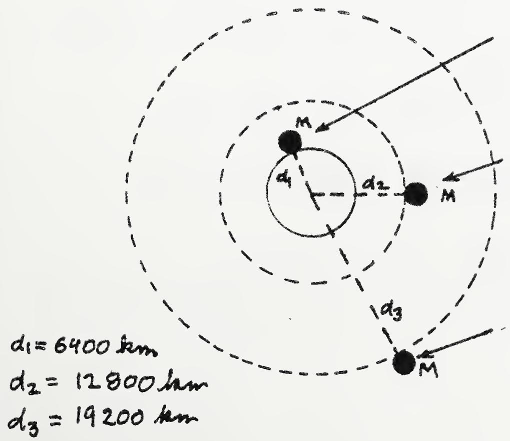
and is called "the gravitational constant".

For this course you will not have to use the formula

$$F = G \frac{m_1 m_2}{d^2}$$

However you should be familiar with the kinds of variations of gravitational attraction shown in the figure below.

Fig. A The effect of distance change on gravitational force.



This object is attracted to the earth with a force of 100 N at the earth's surface which is 6400 km from its center.

The same object at this point is two times as far away from the center of the earth. The force is $(1/2)^2$ or 1/4 as much and is therefore equal to 25 N.

The object is now 3 times as far away, and the force is therefore $(1/3)^2$ or 1/9 as much as it was originally – approximately 11 N.

Study the situation shown in Fig. A. Since the mass remains the same (M), the change in the attraction toward the earth depends upon the fact that the distance between M and the earth changes. Since the force of gravitational attraction varies inversely as the square of the distance between the two bodies, we may write

$$F \propto \frac{1}{d^2}$$

and

$$F = k\left(\frac{1}{d^2}\right) ;$$

where F = force of gravitational attraction,
d = distance between the bodies,
k = a constant.

If the force of gravitational attraction on an object at a distance, d_1, from the earth's center is F_1, and the force of gravitational attraction on the same object a distance, d_2, from the earth's center is F_2, we have

$$F_1 = k\left(\frac{1}{{d_1}^2}\right), \text{ and } F_2 = k\left(\frac{1}{{d_2}^2}\right).$$

Finding the ratio between the two forces, we have

$$\frac{F_1}{F_2} = \frac{k\left(\frac{1}{{d_1}^2}\right)}{k\left(\frac{1}{{d_2}^2}\right)} = \frac{(1/{d_1}^2)}{(1/{d_2}^2)} = \frac{1}{(d_1)^2} \times \frac{1}{\frac{1}{(d_2)^2}}$$

$$= \frac{1}{{d_1}^2} \times d_2^2 = \frac{(d_2)^2}{(d_1)^2} = \left(\frac{d_2}{d_1}\right)^2$$

If F_1 is the weight of the object at the earth's surface, then d_1 is the radius of the earth (6400 km). If d_2 is known, we can find F_2:

$$\frac{F_1}{F_2} = \frac{\text{surface weight}}{F_2} = \left(\frac{d_2}{d_1}\right)^2$$

with F_W = surface weight

$$F_2 = \frac{F_W}{\left(\frac{d_2}{d_1}\right)^2} = F_W\left(\frac{d_1}{d_2}\right)^2$$

Assuming d_1 = radius of the earth = 6400 km, and that d_2 is in kilometres, we have

$$F_2 = F_W\left(\frac{6400 \text{ km}}{d_2}\right)^2$$

We may apply this to the example shown in Fig. A, page 11.

When d_2 = 12 800 km, F_W = 100 N, and F_2 (weight 6400 km above the surface = $F_W\left(\frac{6400 \text{ km}}{d_2}\right)^2$; then

$$F_2 = 100 \text{ N}\left(\frac{6400 \text{ km}}{12\ 800 \text{ km}}\right)^2$$

$$= 100 \text{ N}\left(\frac{1}{2}\right)^2 = 100 \text{ N}\left(\frac{1}{4}\right)$$

$$= 25 \text{ N}.$$

The gravitational attraction varies directly with the product of the masses of the two bodies involved:

$$F \propto m\, M_E$$

where m = mass of the body, M_E = mass of the earth.

Using a constant,

$$F = k\, m\, M_E$$

If we have two masses, m_1 and m_2 both at a distance d from the center of the earth, the two gravitational attraction forces are:

$$F_1 = k\, m_1\, M_E \text{ and } F_2 = k\, m_2\, M_E;$$

where M_E = mass of the earth.

We can find the ratio of F_1 to F_2 as:

$$\frac{F_1}{F_2} = \frac{k\, m_1\, M_E}{k\, m_2\, M_E} = \frac{m_1}{m_2}$$

The ratio of gravitational forces indicates that if two bodies are the same distance from the earth's center, the ratio of attractive forces is equal to the ratio of their masses. For example, if at the surface of the earth a body of mass 1.0×10^2 kg has a weight of 9.8×10^2 N, and another body has a mass of 3.0×10^2 kg, we can find the weight of the second body as follows:

$$\frac{F_1}{F_2} = \frac{m_1}{m_2}, \quad \frac{F_2}{F_1} = \frac{m_2}{m_1}$$

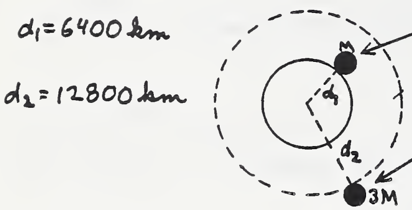
$$F_2 = F_1 \times \frac{m_2}{m_1} = 9.8 \times 10^2 \text{ N} \times \frac{3.0 \times 10^2 \text{ kg}}{1.0 \times 10^2 \text{ kg}}$$

$$= 2.9 \times 10^3 \text{ N}.$$

The weight of the second body is 2.9×10^3 N.

We now have two expressions in which the ratio of gravitational forces may be found for (1) a body of constant mass which changes its position with respect to the earth's center, and (2) two bodies of different masses which are the same distance from the earth's center. We may expand these developments to the case of a body which changes both its position and mass with respect to the earth. See Fig. B.

Fig. B The combined effect of distance and mass change



The force acting on mass M at the surface of the earth is 20 N.

This mass is 3 times as great, so the force is multiplied by 3. However, the mass is also 2 times as far away. Therefore the force is divided by 2^2 or 4. The combined effect of the change in mass and distance is to make the force 3/4 times as great, or 15 N.

The expression for this case is

$$\frac{F_1}{F_2} = \frac{m_1}{m_2} \times \left(\frac{d_2}{d_1}\right)^2$$

OR

$$F_2 = F_1\left(\frac{m_2}{m_1}\right)\left(\frac{d_1}{d_2}\right)^2$$

> Sometimes R is used in place of d to indicate the distance between the centres of two masses..
>
> eg $\quad Fg = \dfrac{Gm_1 m_2}{R^2}$

In this case:

F_1 = original gravitational force
F_2 = new gravitational force
m_1 = original mass
m_2 = new mass
d_1 = original distance from the center
d_2 = new distance from the center

As an example, consider a body which moves from the earth's surface to a distance of 6400 km from the surface, and which has its mass changed to 3 times its original mass. If its original weight was 20 N its new weight may be calculated as follows:

$F_1 = 20\text{ N},\ m_1 = m,\ d_1 = 6400\text{ km}$

F_2 is unknown, $m_2 = 3\text{ m},\ d_2 = 12\ 800\text{ km}$

$$\frac{F_1}{F_2} = \frac{m_1}{m_2}\left(\frac{d_2}{d_1}\right)^2,\quad F_2 = F_1\left(\frac{m_2}{m_1}\right)\left(\frac{d_1}{d_2}\right)^2$$

$$F_2 \doteq 20\text{ N}\left(\frac{3\text{ m}}{\text{m}}\right)\left(\frac{6400\text{ km}}{12\ 800\text{ km}}\right)^2 = 20\text{ N }(3)\left(\frac{1}{2}\right)^2$$

$$= 20\text{ N}(3)\left(\frac{1}{4}\right) = 20\text{ N} \times \left(\frac{3}{4}\right) = 15\text{ N}$$

The new gravitational force is 15 N.

Do the following exercise and send it in for correction.

EXERCISE

1. You have learned that weight and mass are not identical.

   (a) Which one is a vector? ____________________

   (b) Which is independent of position? ____________________

2. At any given position on the earth's surface you double the mass of a body. What then happens to its weight?

   ____

3. Express the following statement in mathematical form (as a proportionality using F, m_1, m_2): the force of gravitational attraction between two bodies is directly proportional to the product of the masses ($m_1 m_2$).

4. Express the following statement in mathematical form: the force of gravitational attraction between two bodies varies inversely as the square of the distance between the two bodies (**d^2**).

5. Study Fig. A on page 11 of the lesson. According to the diagram, from what point in the earth is the distance between the earth's mass and the body of mass **M** or **3 M** measured?

____

Approximately, what is the distance between that point and the earth's surface (give the value shown in the diagrams)?

____

6. Study Fig. A on page 11 of the lesson.

If the object of mass **M** were 3200 km above the earth's surface, it would be __________ km from the earth's center. This means it would be $\frac{9600\text{ km}}{6400\text{ km}}$ or 1.5 times the earth's radius from the earth's center.

If the attraction at the earth's surface is 100 N, and using the idea that the force of gravitational attraction varies inversely as the square of the distance between the two bodies, the force of attraction at 3200 km would be $\frac{1}{(1.5)^2} \times 100\text{ N} =$ __________.

Similarly, a body which is 9600 km above the earth's surface would have a position from the center of the earth that would be $\frac{(\quad\quad)}{6400\text{ km}} =$ _____ times the earth's radius.

Thus if the body were attracted to the earth with a force of 100 N when at the surface, it would be attracted with a force of $\frac{1}{(\quad\quad)^2} \times 100\text{ N} =$ __________ when 9600 km above the surface.

7. Now study Fig. B on page 14 of the lesson. Note that the effect of **both** mass and distance is considered in Fig. B.

From question 6 above, if a body of mass **M** is 9600 km above the surface, it would be attracted by a force that would be $\frac{1}{(\quad\quad)^2}$ times the force of attraction at the surface.

However, if the mass were increased four times to **4 M**, the force of attraction experienced by the mass of **4 M** would be $\frac{4\text{ M}}{\text{M}} = 4$ times the force it would experience at the surface.

If the surface force were 20 N, the force at 9600 km above the surface would be $\frac{4}{(\quad\quad)^2} \times 20\text{ N} =$ __________.

8. Suppose an animal used in a space-flight test weighs 50 N at the earth's surface. What would have been the minimum weight of the animal if it reached a maximum height of 1600 km above the earth's surface?

$$\frac{\text{Distance of animal from earth's centre}}{\textbf{Radius of earth}} = \frac{(\qquad\qquad)}{(\qquad\qquad)} = \frac{(\qquad)}{\qquad}$$

Gravitational attraction varies as $\frac{1}{(\text{distance comparison})^2}$.

∴ Attraction 1600 km above the surface is

$$\frac{1}{(\underline{\qquad\qquad})^2} \times \text{attraction at the surface}$$

$$= \frac{1}{(\underline{\qquad\qquad})^2} \times 50\ \text{N} = \underline{\qquad\qquad\qquad}.$$

9. A rocket initially weighing 6.0×10^6 N is sent into orbit 3200 km from the earth's surface. After using all its fuel, and dropping a number of stages, the space vehicle (in orbit) has a mass of one-tenth of what it had on earth. What force of attraction to the earth would be experienced by the rocket while in orbit? (See expressions on page 14 of the lesson).

Lesson Summary

Newton's Second Law: An unbalanced force acting on an object produces an acceleration in the direction of the force. The acceleration varies directly as the force and inversely as the mass of the object i.e.

$$\overline{F} = m\,\overline{a}$$

Unit of force is the newton. When a newton is applied to a body of mass of 1 kg, the body's acceleration will be 1 m/s^2.

Newton's Third Law states that action and reaction forces are equal and opposite and act on different bodies.

Weight of a body is equal to the magnitude of gravitational force acting on it. i.e. $\overline{F}_w = m\,\overline{a}_g$

Newton's Law of Gravitation states that any two bodies with masses m_1 and m_2 at a distance 'd' apart attract each other with a force which is directly proportional to the product of the masses and inversely proportional to the square of the distance between them. Mathematically stating

$F = G\frac{m_1 m_2}{d^2}$, where $G = 6.67 \times 10^{-11} \frac{N\cdot m^2}{kg^2}$ and is called the gravitational constant.

End of Lesson E

PHYSICS UPGRADING
LESSON F

DO NOT send this lesson for correction

MOMENTUM, ENERGY, WORK, AND POWER

Conservation of Mass

Does something ever disappear without a trace? Do objects ever appear in empty space where previously there was no matter? Common experience seems to indicate that neither of these happen. Yet how can we be sure?

Early in the beginnings of modern science, Sir Isaac Newton discovered a concept by which we can measure quantity of matter. His second law of motion states that **the ratio of force to acceleration of a body (F/a) is always constant.** This quantity we call **mass.**

A famous chemist, Lavoisier, discovered that chemical reactions do not change the masses of the substances involved even if there was burning, emission of light or heat or even explosion. If you burn a substance and carefully collect or preserve all the products you will find that the mass of the products equals the original mass plus the mass of oxygen used for burning. This led to the **law of conservation of mass.** In any closed system the mass of the system remains constant.

Self-Check Exercise #1

1. Which of the following is the **most obvious** evidence that the amount of matter remains constant?

   (a) A rock cliff wearing away into sand on the beach.
   (b) A water puddle drying in the sun.
   (c) The slow disappearance of a moth ball.
   (d) The appearance of the morning dew on grass that was completely dry the evening before. ______

2. What is the standard by which we define mass?

   (a) The volume or space occupied by an object.
   (b) The ratio of force to the acceleration produced in a body by that force.
   (c) The extent to which a body resists compression.
   (d) The volume occupied by the matter. ______

3. What is the process by which we usually measure the mass of a body?

   (a) By noting the effects of the earth's gravitational force on it, that is, its weight.
   (b) By observing the volume that it occupies.
   (c) By analyzing its chemical composition.
   (d) By accelerating it with a given constant force. ______

You will find the answers to these questions on page 3.

The Search for a Quantity of Motion That is Conserved

When a charge of dynamite explodes in a face of rock, particles and dust fly furiously in all directions. But it is not long before the dust and rubble settle down and become motionless. If you drop a hard rubber ball on concrete it will bounce a number of times but eventually it too settles to rest. The motion has disappeared. Or has it?

After the discovery of the law of conservation of mass it was natural to ask whether there is a quantity of motion that is conserved. Careful experiments in isolated systems (an isolated system is one in which there is no net force acting from outside the system) show that the quantity, mass times velocity (**mv**), is conserved. This quantity **mxv** is called **momentum** and it is sometimes represented by the symbol $\vec{p}$. Thus, $\vec{p} = m\vec{v}$. Note that $\vec{p}$ is a vector because $\vec{v}$ is a vector.

The **law of conservation of momentum** states that the total momentum of all the parts before an interaction (such as a collision) is the same as their total momentum after the interaction.

Total momentum before = Total momentum after

If there are two bodies then

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$$

where

$\vec{p}_1$ = momentum of body 1 before the interaction
$\vec{p}_2$ = momentum of body 2 before the interaction
$\vec{p}_1'$ = momentum of body 1 after the interaction
$\vec{p}_2'$ = momentum of body 2 after the interaction

We can also say that, since total momentum is unchanged, the momentum gained by one body must equal the momentum **lost** by another. The total **change of momentum** is zero.

Thus

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$
$$\Delta\vec{p}_1 = -\Delta\vec{p}_2$$

where

$\vec{p}_1$ = change in momentum of body 1
$\vec{p}_2$ = change in momentum of body 2

Self-Check Exercise 2

1. Find the momentum of a 0.14 kg baseball thrown at 30 m/s.

2. What is the **change** in momentum of a 1000 kg automobile that slows from 10.0 m/s to 5.0 m/s?

3. A 0.20 kg arrow is shot horizontally into a 1.0 kg block of wood (initially at rest) which is free to move on a frictionless surface. The initial velocity of the arrow was 60 m/s. If the arrow stays embedded in the wood, calculate the velocity of the arrow and wood after impact.

Answers to these problems can be found on page 4. Be sure that you attempt to do them before you look at the answers.

Answers to Self-Check #1, page 1		
1. a	2. b	3. a

Various Kinds of Collisions

If you place a single billiard ball in the centre of the table and strike it directly on with another billiard ball the second ball will stop dead while the first one goes off with the same velocity as the second one had before they struck. Why? Why do the balls not both have the same velocity after collision?

Christian Huygens, a famous Dutch physicist, suggested that there is another quantity of motion that is **sometimes** conserved. It is the sum of the $\mathbf{mv^2}$ products of all the objects involved. If we take half of that product to obtain $\frac{1}{2}\mathbf{mv^2}$, we will have an expression for what is known as **kinetic energy (KE).**

Momentum is always conserved in a collision, but kinetic energy is not always conserved. Any collision where KE is conserved is called an **elastic** collision. Any collision where KE is less after the collision than before is called **inelastic.** Any collision where KE is totally lost (that is, the objects all come completely to rest) is called **completely inelastic.**

For example, if a ball drops to the floor and bounces back to the same height from which it came, the collision is elastic. If it comes back only part way it is inelastic. If it hits the floor with a plop and sticks there (as a mud ball would) the collision is completely inelastic.

Self-Check Exercise #3

1. What is the momentum of a 2.4 kg ball that is moving at 15 cm/s right? (Change cm/s to m/s).

2. What is the kinetic energy of the ball in question 1?

3. If the ball strikes another object (initially at rest) inelastically how much KE do the ball and object have together? (Give, not a specific value, but a range of values which they could have.)

Answers on page 6.

Answers to Self-Check #2, pages 2 and 3

1. $P = m\overline{v} = 4.2$ kg m/s   2. -5.0×10^3 kg m/s

3. Momentum after = momentum before

   $(1.0 \text{ kg} + 0.20 \text{ kg})v = 0.20 \text{ kg } (60 \text{ m/s})$

   $v = 10$ m/s

Energy

There are probably few ideas in the history of science that are as interesting and fascinating as the idea of energy. As so often is the case, scientists made advances in this area, not so much by doing experiments and making observations but by changing their thinking. Sometimes revolutions take place as much inside the minds of people as they do in the external circumstances. And so it was with energy.

What is energy? We take it so much for granted in our world that we forget how elusive a concept it can be. Energy is not a piece of matter (or is it?) that we can see and touch and weigh. It is almost ghostly with no apparent substance or existence of its own. The fact is that we understand and define energy solely by the results or effects it produces on objects (masses) that we can see and touch and (sometimes) hear. Energy, we say, is whatever there is that can squeeze an object into a new shape, or move an object from here to there when otherwise it would stay put, or accelerate an object to a new vigor of motion. Energy is the property of something that enables it **to do work.** In order to keep things clear and straightforward scientists consider it important to define work. Work is done whenever a force acts through a displacement in the direction of the force. In symbols **Work = F × d.** Thus if 1 N of force east acts through a displacement of 1 m east the work done = 1 N × 1 m = 1 N·m = 1 J (joule). If however, 1 N east acts on a body and the body moves 1 m south, work done = 1 N × 0 m east = 0. You can see that if you strain at an object with great vigor yet do not move it, by the scientific definition you are doing no work. Energy then is the capability of something to do work. Work is the actual expenditure of energy. (Note that work is defined in terms of the product of force and displacement. Since force and displacement are vector quantities, their "product" (work) is a special kind of product, called a "dot product." We will not study this kind of product. In most of the cases with which we shall deal, the force and the displacement will be in the same direction, and we will be able to calculate work by using just the magnitudes of force and displacement. In some cases, force and displacement will not have the same direction. To calculate work in those cases ,it will be necessary to find the component of displacement that is in the direction of the force. In most cases this can be done by using some trigonometry: finding the cosine of the angle between the force and the displacement, and multiplying it by the magnitude of the displacement. See **Self-Check Exercise #4,** question 3.)

Self-Check Exercise #4

1. What is the work done when an object of 5.0 kg is moved by a force of 2.3 N north through a distance of 1.6 m north?

2. What is the work done by a 70 kg man who climbs a 3.0 m ladder? (Hint: first find the gravitational force on the man — the force he must overcome to climb upward.)

3. What is the work done by a force of 350 N south that moves an object 4.5 m southeast? (Hint: find the component of the displacement that is in the southerly direction.)

Answers can be found on page 8.

Answers to Self-Check #3, page 4

1. 0.36 kg·m/s   2. $KE = \frac{1}{2}mv^2$ $2.7 \times 10^{-2}\ \frac{kg \cdot m^2}{s^2}$

3. Between 0 and $2.7 \times 10^{-2}\ \frac{kg \cdot m^2}{s^2}$.

How Fast Can You Climb Stairs?

Sometimes it is important for us to know not only whether an object can be moved from place to place but how fast it can be moved. If you were to climb the stairs of the AGT building or the Calgary Tower at a normal pace it would require perhaps five minutes. By elevator, however, it would likely take less than one minute. We say that the elevator produces more **power** than the person's muscles. Power is defined as the rate of doing work. In algebraic form:

$$\text{Power} = P = \frac{\text{Work}}{\text{Time}} = \frac{Fd}{t}$$

The units of power are: $\frac{N \cdot m}{s} = \frac{J}{s} = W$ (watts).

Take an example:

How much power does a person with a mass of 70 kg develop when he climbs stairs to the top of a 100 m building in 300 s?

$$P = \frac{Fd}{t} = \frac{F_W d}{t} = \frac{ma_g d}{t} = \frac{(70\ kg \times 9.8\ m/s^2) \times 100\ m}{300\ s}$$

$$= 229\ W = 2.3 \times 10^2\ W = 0.23\ kW$$

Note that this is almost the power that would be given off by four 60 W light bulbs.

If we note that $\frac{d}{t} = v$ (velocity) we can also write the power equation as $P = Fv$. Finally we observe that work or energy can be found by multiplying power by time. Work = Pt

Self-Check Exercise #5

1. What is the power developed by an engine that lifts 25 kg a vertical distance of 10.0 m in 2.00 min?

2. What is the power developed by a grader whose blade exerts a force of 3.0×10^4 N as it moves at 0.80 m/s?

3. An automobile with a power output of 12 kW exerts a force of 480 N as it moves along the highway at constant velocity. How fast is the automobile moving? (Find the answer in both m/s and km/h).

Answers on page 12.

Lesson Summary

The law of conservation of mass states that in any interaction of **substances, physical or chemical, the amount of mass remains constant.**

The momentum of an object is defined as **the product of its mass and its velocity:**

$$\vec{p} = m\vec{v}$$

In any collision or interaction of objects momentum is the quantity of motion that is always constant. This is the law of the conservation of momentum.

Another quantity of motion that is sometimes conserved is the product mv^2. We can determine that the amount of work that can be done by a moving body if it gives up all its motion is $\frac{1}{2}mv^2$. This is called the **kinetic energy** of a body.

If kinetic energy (KE) is conserved in a collision or interaction, the collision is described as **elastic**. If KE is not conserved the collision is **inelastic**.

Energy is the capability of doing work. Work is defined as the product of a force and the distance through which it moves:

$$W = Fd$$

Power is the rate of doing work:

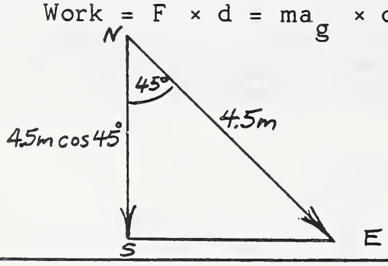
$$P = \frac{Fd}{t} \quad \text{or} \quad P = Fv$$

Answers to Self-Check Exercise #4, pages 5 and 6.

1. 3.7 J. Note here that the 5.0 kg does not enter into the problem. We are only interested in the force acting through the distance.

2. Work = F × d = ma_g × d = (70 kg × 9.8 m/s²) × 3.0 m = 2.1 × 10³ J.

3.



Work = 350 N × 3.18 m
= 1.1 × 10³ J

Exercises

1. Which of the following is an expression for work?
   (a) Fv
   (b) P/F
   (c) Ft
   (d) $ma_g d$
   ______

2. What must you multiply F by to get an expression for power?
   (a) $\frac{1}{t}$
   (b) v
   (c) ma_g
   (d) d
   ______

3. Which of the following quantities of motion is conserved in an inelastic collision?

   (a) **mv**
   (b) **mv²**
   (c) **½mv²**
   (d) **v**

   ______

4. If 50 mL of alcohol are mixed with 50 mL of water, the mixture amounts to 98 mL. Is this a contradiction of the law of conservation of mass? Explain.

   ____

   ____

5. What is your momentum while you are walking at 1.4 m/s south?

6. A football player of mass 100 kg has a momentum of 800 kg•m/s to the right. What is his velocity? (Remember that velocity includes direction.)

7. An empty railway car of mass 5000 kg moves down a level track with negligible friction at 2.4 m/s. As it passes a coal chute 2500 kg of coal are dumped into it. What is the speed of the car after passing the chute?

8. (a) A 14.0 kg ball moving at 14.0 m/s approaches a 12.0 kg ball moving at 10.0 m/s. At collision the 14.0 kg ball slows to 11.0 m/s. What is the velocity of the 12.0 kg ball after collision? (Note in this question that all of the velocities are in the same direction. If the 14.0 kg ball had bounced **back** its velocity would have been **negative.**)

(b) What is the total kinetic **energy** of the two balls **before** collision?

(c) What is the total kinetic energy of the two balls **after** collision?

(d) Is the collision elastic or inelastic? ____________________

Why? ____________________

(e) What do you think has happened to the difference between the energy before and the energy after?

____

____

9. How much work is done when a 15.0 N force moves an object 3.8 m?

10. Lifting an object 3.2 m requires 75 J of work. What is the mass of the object? (Assume that the body is lifted at constant slow velocity.)

11. A 500 N hammer blow drives a nail 3 mm into a board. How much work is done?

12. How much power does it take to do 5.0×10^2 J of work in 8.0 s?

13. What is the power developed by a man who lifts 30 bales of hay, each of mass 15 kg, a distance of 1.0 m in 1.0 min?

14. What is the power delivered by a locomotive that exerts a force of 9.00×10^4 N to pull a train at 1.00×10^2 km/h? (27.8 m/s)?

15. How much energy is consumed when an 800 W dryer element in an electric dishwasher operates for 10 min? (Give your answer in **both** watt-hours and joules.)

16. How long does it take a 40 kW engine to do 100 000 J of work?

Answers to Self-Check Exercise #5, page 7.

1. $$P = \frac{Fd}{t} = \frac{F_W d}{t} = \frac{ma_g d}{t} = \frac{(25\ \text{kg} \times 9.8\ \text{m/s}^2)\ 10\ \text{m}}{120\ \text{s}} = 20.4\ \text{W} = 20\ \text{W}$$

2. $$P = Fv = (3.0 \times 10^4\ \text{N}) \times 0.80\ \text{m/s} = 24\ 000\ \text{N}\cdot\text{m/s} = 2.4 \times 10^4\ \text{W} = 24\ \text{kW}$$

3. $$P = Fv$$

$$v = \frac{P}{F} = \frac{12\ 000\ \text{W}}{480\ \text{N}} = 25\ \text{m/s}$$

$$25\ \text{m/s} \times 3600\ \text{s/h} \times \frac{1\ \text{km}}{1000\ \text{m}} = 90\ \text{km/h}$$

(*Note how units cancel:* $\frac{\cancel{m}}{\cancel{s}} \times \frac{\cancel{s}}{h} \times \frac{km}{\cancel{m}} = \frac{km}{h}$)

End of Lesson F

PHYSICS UPGRADING
LESSON G

DO NOT send this lesson for correction

KINETIC ENERGY AND POTENTIAL ENERGY

Kinetic energy is the energy (or capability of doing work) that any body possesses **because of its motion.** Any moving object has kinetic energy. We can easily show that its energy is $\frac{1}{2}mv^2$ if we remember that it requires a force to accelerate a body from rest ($v = 0$) to some velocity v.

Suppose a constant force F is applied to a body of mass m giving it a constant acceleration, a. We know that in time t a body constantly accelerating at a will cover a distance of $d = \frac{1}{2}at^2$. But we also know that the velocity $v = at$ and so $t = \frac{v}{a}$. Therefore $d = \frac{1}{2}a\left(\frac{v}{a}\right)^2 = \frac{1}{2}a\frac{v^2}{a^2} = \frac{1}{2}v^2/a$.

Now the work done when a force, F acts through a distance, d is

$$\text{Work} = Fd$$

Here the work done

$$= F \times d$$

$$= ma \times \frac{1}{2}\frac{v^2}{a}$$

$$= \frac{1}{2}mv^2$$

Thus the body has acquired an energy of $\frac{1}{2}mv^2$ as a result of the work done on it. Notice that the KE is not dependent at all on a. It depends only on the mass and the velocity. It does not matter how fast that velocity was achieved.

Self-Check Exercise #1

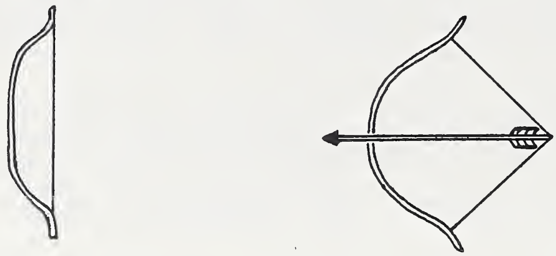
1. What is the KE of a 1000 kg automobile travelling at 90 km/h? (Note: change km/h to m/s.)

2. What force must be applied to the automobile if it is slowed down to 0 from 90 km/h in 10 s?

Answers on page 4. Be sure to attempt to solve the problems on your own before checking the answers.

Potential Energy

A bow consists of a long narrow piece of wood or metal across the end of which a string is attached. An arrow is fitted to the string, the bow is



held in one hand and the string pulled back with the other. In order to pull the string back the string hand must exert a force through a distance. This means that it does work. What happens to that work? When the bow is fully stretched there is no motion so no KE is involved. There must be energy stored in the bow. We have evidence that this is so because when the string is let go it pushes against the end of the arrow and accelerates it to a high velocity. It does work on the arrow because it pushes the arrow through a distance. The energy stored in the bow is changed to kinetic energy in the arrow. The energy in the bow is called potential energy — potential because it is stored in the bow because of its stretched **position.**

One can also produce stored energy by stretching or compressing a spring (as in an air gun), by stretching or twisting a rubber band (as in a model airplane) or by pulling magnets apart. Energy can be stored by lifting an object above the ground against the force of earth's gravity (as in a piledriver). In all these cases potential energy is created due to the **position** or **condition** of the mass involved. KE on the other hand is solely due to the **motion** of a mass.

Self-Check Exercise #2

1. A bow requires a force of from 0 to 200 N as it is stretched from rest position to its fully extended position. Its arrow is 80 cm long. How much energy is stored in the bow when it is ready to shoot the arrow? (Hint: Use the **average** force required to stretch the bow to find the energy.)

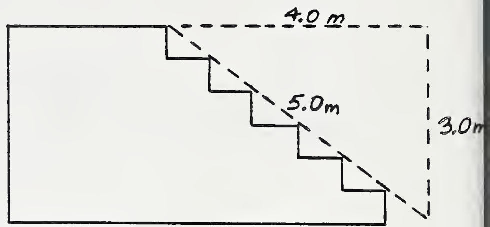
2. Assume that all the energy stored in the bow is transferred to the arrow. What KE does the arrow have just as it leaves the bow?

3. If the mass of the arrow is 100 g what is the velocity of the arrow?

4. (a) How much potential energy is stored in the body of a 75 kg man who climbs to the sixth floor of a building that has a distance of 3.0 m between floors? (Hint: be careful in finding the total distance — use a diagram.)

   (b) How does this potential energy compare with his kinetic energy as he travels on a bicycle at 14 m/s?

5. A 100 kg refrigerator is moved 5.0 m up along a stairway as shown in the diagram. What is the potential energy of the refrigerator at the top of the stairway? (Hint: Note that it is the **vertical height** that must be considered since the force of gravity acts directly **downward.** It does not require much force to move a refrigerator horizontally if it is on rollers to reduce friction.)



Answers on page 7.

Answers to Self-Check #1 , page 1
1. $KE = \frac{1}{2} m\overline{v}^2 = 3.1 \times 10^5$ J   2. $F = m \frac{\Delta v}{\Delta t} = -2.5 \times 10^3$ N

The Relativity of Potential Energy and Kinetic Energy

In order to calculate KE of a body we must know the velocity of the body. Usually we assume that we are measuring velocity with respect to the surface of the earth. Yet that need not be so. For example when you are flying in an airplane at 240 m/s your kinetic energy **with respect to the earth** is $\frac{1}{2}(m)(240\ m/s)^2$ but **with respect to the plane** your KE is 0 because you are not moving with respect to the airplane — you are sitting in your seat.

Likewise with potential energy. If you are on the sixth floor of a building your potential energy with respect to the earth is ma_gh (where h is your height above earth) but your PE with respect to the sixth floor is zero or nearly zero because you are standing on it.

Thus when we calculate KE or PE we must either clearly assume that the earth's surface is the frame of reference or we must specify what frame of reference we are using.

Conservation of Mechanical Energy

In ordinary usage "to conserve" means to save something or not to waste something. In this lesson, when we talk about conservation of mechanical energy, we are not concerned so much with saving the energy, but rather with accounting for all of the energy that might be involved in some process or event. Usually, in physics the phrase "conservation of energy" refers to the idea that it is possible to explain what has happened to the energy involved in a physical interaction.

A Falling Body

What happens when a body falls from a certain height to the surface of the earth?

We know first of all that before it falls its potential energy with respect to the ground is $ma_g h$ where h is its height above the ground.

We also know that as it falls it has a constant acceleration a_g (neglecting air friction) as the earth's force of gravity acts on it. Its velocity gradually increases until it hits the ground.

What will be its kinetic energy just as it strikes the ground? Well we know that $KE = \frac{1}{2}mv^2$. But also we know that $h = \frac{1}{2}a_g t^2$ where t is the time of the fall and that $v = a_g t$ or $t = \frac{v}{a_g}$. Putting these two together we have $h = \frac{1}{2}a_g\left(\frac{v}{a_g}\right)^2 = \frac{1}{2}a_g \times \frac{v^2}{a_g^2} = \frac{1}{2}\frac{v^2}{a_g}$. We could also write this as $v^2 = 2a_g h$.

Now $KE = \frac{1}{2}mv^2 = \frac{1}{2}m(2a_g h) = ma_g h$. But $ma_g h$ is the original potential energy.

The mathematics seems to be telling us that the KE at the ground equals the PE before it falls. Apparently what the body has lost in PE it has gained in KE. Indeed if we examine the body at every point along its path we will find that whatever is lost as PE is gained as KE. Another way of saying this is that **KE + PE = a constant.**

Self-Check Exercise #3

1. A body of mass 409 g rests on a ledge 20 m above the ground? What is its potential energy?

2. What is the potential energy after it has fallen 5.0 m?

3. What do you think its KE should be after it has fallen 5.0 m?

4. Find its velocity (using $v^2 = 2a_g h$) after it has fallen 5.0 m.

5. Now find its KE at this velocity.

6. How does this KE compare to your guess in #3?

7. What is its KE + PE after it has fallen 5.0 m?

8. How does this compare with the PE initially?

Answers on page 12.

Any system in which the KE + PE of an object is constant is called a **conservative** system. In any such system there is a **conservation of mechanical energy.**

Another example in which conservation of energy may apply is when an object is given PE and then falls to the earth attaining maximum KE, then drives something through a distance. In this case work is done on the object by the KE which comes from the original PE.

Example: A 100 kg pile driver is lifted to a height of 3.06 m. It falls and drives a post 10 cm into the ground. Find the average force on the post.

$$PE = 100\ kg \times 9.8\ m/s^2 \times 3.06\ m = 3.0 \times 10^3\ J$$

$$F \times d = Work = PE$$

$$F = \frac{PE}{d} = \frac{3.0 \times 10^3\ J}{0.10\ m} = 3.0 \times 10^4\ N$$

Lesson Summary

Kinetic energy is the energy that a body possesses due to its motion. It is calculated from $KE = \frac{1}{2}mv^2$. We know that a moving object has KE because it takes a definite force acting through a distance to accelerate it to the velocity it has acquired.

Potential energy is the energy that a body possesses due to its position or condition. It can be found by multiplying an average force required to move the body from some initial condition to some final condition by the distance through which the force acts: $PE = F \times d$.

Both PE and KE are relative quantities because they are both calculated with respect to some frame of reference.

A conservative system is one in which KE + PE is a constant. In such a system there is conservation of mechanical energy.

Answers to Self-Check #2, pages 2, 3 and 4.

1. $PE = F_{av}d = 80\ J$ 2. $80\ J$ 3. $v^2 = \frac{KE}{\frac{1}{2}m}$

$$= \frac{80\ J}{\frac{1}{2}(0.100\ kg)} = \frac{80\ kgm^2/s^2}{0.050\ kg} = 1600\ m^2/s^2;\ v = \sqrt{1600\ m^2/s^2} = 40\ m/s$$

4. (a) $PE = ma_gh = 1.1 \times 10^4\ J$ (b) $KE = 7350\ J = 7.4 \times 10^3\ J$ PE is more

5. $2940\ J = 2.9 \times 10^3\ J$

Exercises

1. (a) What is the kinetic energy of a 1 g bug flying at 2 m/s?
   (b) What is his kinetic energy if his mass doubles because he's carrying something?
   (c) What is his kinetic energy if he's carrying nothing but doubles his speed?

2. A force is exerted on a 10 kg object for 3.0 m, on a frictionless surface, giving the object a velocity of 20 m/s. Find:
   (a) the kinetic energy of the object and
   (b) the force that acted on the object.

3. A 200 kg iceboat is pushed 36 m across a frozen lake by a wind of average force 100 N. Assume that the frictional forces are negligible and the boat starts from rest. Find the speed of the boat after 36 m using each of the following methods:

   (a) Using Newton's second law to find the acceleration of the boat. How long does it take to move 36 m? How fast will it be moving by then?

(b) Find the final speed of the boat by equating the work done on it by the wind and the increase in its kinetic energy. (Use $F \times d = \frac{1}{2} mv^2$. Assume v is unknown. Solve for v.)

4. How does potential energy differ from kinetic energy?

5. Describe three different situations or systems that have potential energy.

6. (a) Find the KE of 80 kg sprinter who does the 100 m dash in 10.0 s. Assume the final velocity is the sprinter's average velocity.

(b) How high would the sprinter have to climb so that his PE would equal the KE he has during a sprint. (That is, if all his KE could be converted to high jump energy how high could he jump?)

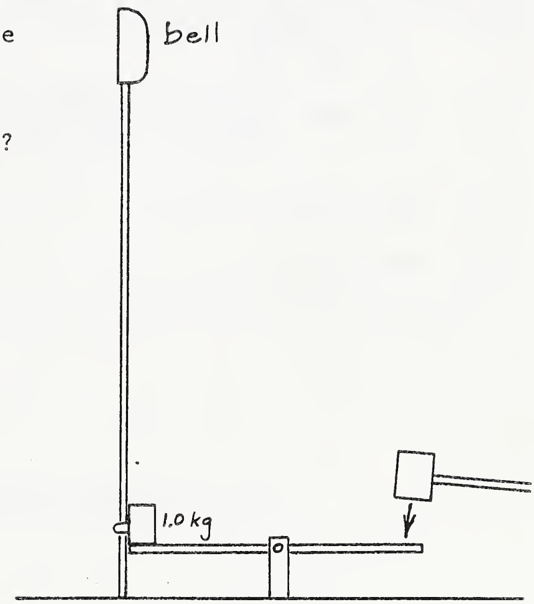
7. A 20 kg object is 2.0 m above the ground. What is its potential energy?

8. It takes 200 J to push an object to the top of a frictionless ramp. What is the potential energy of the object? How much work can it do in sliding down the ramp?

9. (a) A spaceship, the Ambrosia, is in an orbit around the earth 1600 km above the earth. Another spaceship, the Olympia, is in a lower orbit at 900 km above the earth's surface. How much energy would it take to boost a 200 kg rocket containing a 75.0 kg man from Olympia to Ambrosia? (Note: the mean value of $\mathbf{a}_\mathbf{g}$ between 900 km and 1600 km above the earth's surface is 6.86 m/s^2). Express distances in appropriate units.

(b) If the boosting were to be done in three hours how much power would the rocket engine need to produce?

10. (a) At the fairgrounds is a hammer swinging contest where a 1.0 kg object is accelerated upward along a pole toward a bell at the top of the pole. The object is accelerated upward by a blow from a 10 kg hammer that strikes a bar pivoted at its centre. The bell is 3.2 m above the resting place for the 1 kg weight. At least how fast must the object be travelling at the bottom in order to hit the bell at the top?



(b) At what point along the pole will the 1.0 kg object have the most PE after the hammer strikes?

____

(c) At what point along the pole will the 1.0 kg object have the most KE after the hammer strikes?

____

(d) At what point will the object have half KE and half PE?

____

(e) How much is the KE at the point where PE = KE?

Answers to Self-Check #3, pages 5 and 6

1. $PE = 0.409\ kg \times 9.8\ m/s^2 \times 20\ m = 80\ J$

2. $PE = 0.409\ kg \times 9.8\ m/s^2 \times 15\ m = 60\ J$

4. $v^2 = 2\ a_g h = 2 \times 9.8\ m/s \times 5.0\ m = 98\ m^2/s^2$

$v = 9.9\ m/s$

5. $KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.409\ kg)(98\ m^2/s^2)$

$= 20\ J$

7. $20\ J + 60\ J = 80\ J$

8. The same.

End of Lesson G

PHYSICS UPGRADING
LESSON H

DO NOT send this lesson for correction

HEAT, TEMPERATURE AND THE CONSERVATION OF ENERGY

The quantity of motion that is always conserved in an interaction between two or more bodies is called momentum **(mv)**. It is conserved regardless of what else happens. But we have also learned that in many if not most interactions (such as collisions between two objects) the KE is not conserved. What happens to the KE that is lost? We have discovered that KE can be changed to PE and back again. Is it possible that we can somehow account for the loss of KE in a collision? Could there be conservation of energy after all?

An answer to that question begins back in the time of ancient Greece when philosophers were asking questions like:

> **What is the nature of matter?**
> **Can it be divided up into small and smaller pieces forever?**
> **Or does one finally come to a smallest possible piece?**

Two men of Athens, Leucippus and Democritus, about 440 B.C. suggested that matter is made of invisible (and indivisible) particles called atoms. (The word atom means uncuttable). The idea was not popular and died out until the eighteenth century. Then, as a result of observations in chemistry, John Dalton revived the atomic theory of matter.

Today, the idea that matter is made up of very small particles has captured our thinking because it is so useful in explaining and predicting many observations. Its usefulness in physics began already in the 17th century when Leibniz, the famous German philosopher and mathematician suggested that not only is **mv** conserved but so also is $\mathbf{mv^2}$ even in inelastic collisions. The KE that appears to be lost in an inelastic collision is only "dissipated among the small parts." However, macroscopically, we can still say that only in an elastic collision is KE conserved.

Today the kinetic molecular theory of matter, as it is called, makes four claims:

1. All matter is made up of small collections of extremely small particles. The particles are called atoms, and the collections, molecules.

2. Between the molecules are spaces whose size depends on the state of the substance (solid, liquid or gas) and the size of the molecules.

3. The molecules act on all other molecules with forces of attraction.

4. All the molecules of a substance are in more or less rapid motion.

Along with this theory a lot of attention in the 17th century was being given to heat. Some thought heat was a fluid but Count Rumford, an English exile of the late 18th century, discovered that you could get as much heat as you want in boring cannons as long as you continue to put work into it. Then around 1840, J.P. Joule, an amateur English scientist showed that you can get a definite amount of heat from an equally definite amount of mechanical energy. Rapidly it was becoming clear that heat is actually a form of energy. Late in the 18th century several people had suggested that heat is really nothing more than the motion of molecules. And now this idea had experimental support in the work of Rumford and Joule.

Today we think of heat as related to the motion of molecules. Temperature is related to the **average** KE of moving molecules while heat is related to the total KE of all the molecules together.

What then happens when KE is lost in an interaction between two objects? Whenever two substances are in contact with each other there is a force of attraction between them. When the two substances are moved against each other, the molecules are pulled away a bit from their position by this force and then snap back into position. The result is increased motion of the molecules which we sense as heat. A very simple demonstration of that is to rub your hands together. They become warm as the work you do in moving your hands is changed to heat – increased motion of molecules in your hand. An automobile that is coasting we would expect to continue moving indefinitely according to Newton's first law of motion. However it will eventually come to a stop as its KE is changed to heat in the tires and in the road.

The force that results from the attraction between molecules is called **friction.** It is friction that makes it possible for KE to be changed to heat. Thus the amount of KE that seems to disappear in an interaction is actually changed to heat as the substances are deformed or warm up from the interaction. This may happen through friction or simply through direct collision of the molecules as happens in a collision. The result is that the molecular motion increases. The average KE of the molecules becomes greater. Heat energy is added to the bodies and their temperatures rise.

All of this suggests that the energy is not lost but is simply changed in form. This is the law of **conservation of energy.**

Self-Check Exercise #1

1. Which of the following assumptions from the kinetic molecular theory help best to explain **where** lost KE is to be found after an inelastic interaction?

   (a) Matter is made up of very small particles.
   (b) There are spaces between molecules.
   (c) There are forces of attraction between molecules.
   (d) The molecules are in more or less rapid motion. ______

2. Which of the following helps to explain why KE **seems to disappear** in an interaction of two bodies?

   (a) Matter is made up of very small particles.
   (b) There are spaces between molecules.
   (c) There are forces of attraction between molecules.
   (d) The molecules are in more or less rapid motion. ______

3. Which of the following assumptions helps to explain why it is possible to **transfer** energy from KE of a body to heat in the body or surface that it moves over?

   (a) Matter is made up of very small particles.
   (b) There are spaces between molecules.
   (c) There are forces of attraction between molecules.
   (d) The molecules are in more or less rapid motion. ______

4. To which of the following kinetic molecular assumptions is **temperature** most closely related?

   (a) Matter is made up of very small particles.
   (b) There are spaces between molecules.
   (c) There are forces of attraction between molecules.
   (d) The molecules are in more or less rapid motion. ______

5. A flat piece of wood of mass 20 kg moves over a flat wood surface due to application of a steady 80 N force. The force of friction is 70 N and the object moves for 10.0 s. The wood starts from rest.

   (a) What is the accelerating force?

   (b) What is the acceleration?

   (c) How far does the body move in 10.0 s?

   (d) How much energy is lost to the force of friction over this distance?

   (e) How much KE does the body have after 10.0 s?

(f) What is the total energy supplied to the body?

(g) Calculate the total energy expended from the total distance travelled and the total force exerted.

Answers to **Self-Check Exercise #1** on page 6.

Note that in #5 of **Self-Check Exercise #1** most of the energy is lost to friction. Only a bit of it is stored as KE in the moving piece of wood. The frictional force could be greatly reduced if the object were rolling rather than sliding. Much more of the energy could be changed to KE.

Heat and Temperature

Recall the observation that Count Rumford made as he was busy boring cannons. As long as the boring continued, heat was produced in the cannon. As long as mechanical energy is put into the system, heat can be produced. This suggests that heat is a form of energy. If the kinetic molecular theory is accepted, heat is really a form of kinetic energy on a small scale. Leibniz was quite right when he suggested that lost KE is actually "dissipated among the small parts."

J.P. Joule went on to show that a specific amount of heat is produced by a specific amount of mechanical energy. This suggests the idea of the **conservation of energy** something which will occupy our attention a little later.

Friction is one source of heat. It is due to the attractive forces between molecules which can be used to transfer mechanical energy of motion on a large scale to the KE of the molecules on a small scale. As the body moves over a surface, the molecular forces cause the molecules of the surface to vibrate faster, producing heat. Another way to heat a surface is to put it in contact with a hotter surface. In the hotter surface the molecules are moving rapidly. As they strike the slower moving molecules of the cool surface they pass on some of their energy and slow down. The cool surface molecules speed up. Eventually all the molecules have the same average KE and the temperatures of the two bodies are equal. Thus heat can flow between bodies through molecular collisions if the bodies are in contact.

We recall from a previous discussion that heat is related to the total KE of a collection of molecules while temperature is related to the average KE of the individual molecules.

Specific Heat Capacity

One of the striking things that we can discover about substances is that each one has its own heat capacity. We know for example, that metals heat up and carry heat much more rapidly than plastic or glass. Similarly a mass of metal cools off more rapidly than say an equal mass of water. We are not equipped to do an experiment to show that this is so, but if we were, we would find that if we take 1 kg of a number of substances and heat each of them up by 10°C that they absorb widely different quantities of energy. Our results could be summarized as follows:

HEAT ENERGY REQUIRED TO RAISE THE TEMPERATURE OF ONE KILOGRAM OF SELECTED SUBSTANCES BY 10°C

Substance	Energy Required
1 kg - Iron	4 980 J
1 kg - Aluminum	9 090 J
1 kg - Copper	3 900 J
1 kg - Lead	1 280 J
1 kg - Glass	6 700 J
1 kg - Water	41 900 J

Self-Check Exercise #2

1. Which of the above substances requires the **least** energy to change its temperature by 10°C? ____________

2. Which of the above substances requires the **most** energy to change its temperature by 10°C? ____________

3. Suppose you have a source of heat that can provide 3000 J.
   (a) Which of the above substances would have its temperature raised the **most** by 3000 J? ____________
   (b) How high would the temperature go?

Answers to Self-Check #2 on page 7.

Answers to Self-Check Exercise #1, page 2.

1. d 2. a 3. c 4. d

5. (a) 80 N − 70 N = 10 N (b) $a = \frac{F}{m} = \frac{10\ \text{N}}{20\ \text{kg}} = 0.50\ \text{m/s}^2$

(c) $d = \frac{1}{2}at^2 = \frac{1}{2}(0.50\ \text{m/s}^2)(10\ \text{s})^2 = 25\ \text{m}$

(d) $70\ \text{N} \times 25\ \text{m} = 1750\ \text{J} = 1.8 \times 10^3\ \text{J}$

(e) $v = at$ and $KE = \frac{1}{2}mv^2 = \frac{1}{2}m(at)^2 = 250\ \text{J} = 2.5 \times 10^2\ \text{J}$

(f) $1750\ \text{J} + 250\ \text{J} = 2000\ \text{J} = 2.0 \times 10^3\ \text{J}$

(g) $80\ \text{N} \times 25\ \text{m} = 2.0 \times 10^3\ \text{J}$

The results in the table and exercise above show that each substance has a different **heat capacity.** Usually we indicate that fact by assigning a **specific heat capacity.** The specific heat capacity of any substance is the heat required to raise the temperature of **one kilogram** of that substance by **one degree Celsius.** Following is a table showing the specific heat capacities of some well-known substances.

Substance	Specific Heat Capacity (J/kg•°C)
water	4190
alcohol (wood)	2510
aluminum	909
copper	390
gold	132
iron (cast)	498
lead	128
mercury	139
silver	236
glass	670

The definition of specific heat capacity suggests a formula to calculate the total number of joules of energy involved whenever a mass of substance is heated or cooled. It is:

$$h = mc\Delta t$$

Here h is the heat energy (heat absorbed or heat given off), m is the mass of the substance, c is the specific heat capacity and Δt is the temperature change. Thus the amount of heat absorbed by a body depends on its mass, its specific heat capacity and the temperature change.

Self-Check Exercise #3

1. What is the heat lost when a 25 kg pail of water drops in temperature from 40°C to 10°C?

2. What is the mass of a piece of lead that requires 5808 J to heat it from 25°C to its melting temperature 327.5°C?

Answers on page 14.

Answers to Self-Check Exercise #2, page 5.

1. Lead 2. Water 3. Lead

4. $\frac{3000\ J}{1280\ J} \times 10°C = 23.4°C = 23°C$

Heats of Fusion and Vaporization

An interesting thing happens when water is heated to its boiling point. Up to that point every kg of water absorbs 4190 J for every increase in temperature of 1°C. But at 100°C the temperature stops increasing. Each kg absorbs 2.261×10^6 J (2261 kJ) **without changing temperature** as it changes from water to water vapor. This value (2261 kJ) is called the heat of vaporization of water. Similarly when water cools to its freezing temperature 0°C it will lose 4190 J per kg for every decrease in temperature of 1°C. But at 0°C its temperature stops changing even while it loses more heat. Every kg of water loses 3.35×10^5 J (335 kJ) as it changes from water to ice. The opposite happens when ice is melted. 335 kJ of heat must be added to each kg of ice at 0°C to change it to water at 0°C.

Example: How much heat is lost when 2.0 kg of water vapor condense, cool to 0°C and freeze?

$$h = (2.0 \text{ kg} \times 2.261 \times 10^6 \text{ J/kg}) + (2.0 \text{ kg} \times 4190 \text{ J/kg} \times 100^\circ \text{C})$$

$$+ (2.0 \text{ kg} \times 3.35 \times 10^5 \text{ J/kg}) = 4\ 522\ 000 \text{ J} + 838\ 000 \text{ J} + 670\ 000 \text{ J} =$$

$$= 6\ 030\ 000 \text{ J} = 6\ 030 \text{ kJ} = 6.0 \times 10^3 \text{ kJ}.$$

Conservation of Energy

It was discovered in the eighteenth century that heat is a form of energy. That means simply that heat is capable of doing work. That fact was shown clearly in the invention of the steam engine where heat obtained from burning coal was changed to mechanical energy. Thus it became clear that there are two kinds of energy. And if there are two, why not more?

As more and more observations and experiments were made in physics and chemistry it became clear that there were in fact a number of different kinds of energy.

Electrical energy is the energy due to the forces that exist between opposite electrical charges.

Chemical energy is the energy contained in the bonds that form between atoms to form molecules. For example, natural gas molecules are mostly methane, CH_4. When CH_4 burns, oxygen joins with the gas molecules to break them up and energy is released in the form of heat.

Light energy is the energy that comes from rapidly changing magnetic and electric fields that form waves.

Nuclear energy is the energy contained in the forces that hold the nucleus of the atom together.

Mechanical energy is due to either the motions of masses or the positions of masses in a gravitational field.

Energy can change back and forth into any of these different forms but when it does so no energy is lost or gained. The quantity of energy remains constant. We have no absolute proof for this idea of conservation of energy but it has helped to explain and predict so many relationships and events in the physical world that we feel justified in accepting the law with complete confidence.

Some examples of common changes in energy are:

1. Electric light bulb – electrical energy is changed to heat and light energy.
2. Automobile engine – chemical energy is changed to heat energy which is then changed to mechanical energy.
3. Electric generator – mechanical energy (from a turbine or engine) is changed to electrical energy.

In all cases the total energy remains constant – only the form changes.

Lesson Summary

The kinetic molecular theory of matter suggests that heat energy is a form of molecular motion – KE on a very small scale.

Temperature is related to the average KE of each of a collection of molecules while heat is related to the total energy content of the collection.

Energy seems to disappear in some interactions between bodies because there is a change in its form. In large masses energy is often lost to friction.

The law of conservation of energy says that energy may change from one form to another but then the total amount of energy remains constant. This law is extremely important in helping us analyze, understand and study physical happenings that would otherwise be very difficult to analyze.

The specific heat capacity of a substance is the energy absorbed or given off by 1 kg of the substance for every 1°C change in temperature.

Heat lost or gained by a substance is given by $h = mc\Delta t$.

The heat of fusion is the energy required to change a kilogram of the substance from solid to liquid. The heat of fusion of water is 335 kJ/kg or 335 J/g.

The heat of vaporization of a substance is the heat required to change one kilogram of the substance from liquid to vapor. The heat of vaporization of water is 2261 kJ/kg or 2261 J/g.

There are many different forms of energy and one form of energy can be changed to other forms. In doing so the total amount of energy remains constant.

Exercises

1. According to the kinetic molecular theory, what is heat in a substance?

____

____

____

2. What quantity of motion is conserved in an inelastic collision?

____

3. What happens to the KE that is not conserved in an inelastic collision or in a situation where an object moves over a surface?

____

____

____

4. What makes a swinging pendulum come to rest? Where does the energy of the pendulum go?

____

____

____

5. A large balloon carries a man to a particular height above the ground, and then remains stationary at that height. A man having a mass of 80.0 kg jumps from the balloon, and falls for 180 s, reaching a velocity of 63.0 m/s downward.

   (a) Calculate his average acceleration.

(b) Why does the value of the average acceleration differ from the value of $\mathbf{a_g}$ (9.8 m/s^2)?

___

___

___

(c) Calculate the value of his kinetic energy if he had fallen **freely** for 180 s.

(d) Calculate the kinetic energy he actually possesses.

6. What happens to the molecules of a body absorbing heat?

___

___

___

7. What happens to the molecules of a body which is cooling?

____

8. In terms of molecules, what is the difference between temperature and heat?

____

____

____

____

9. (a) What is the heat required to melt a 25 kg block of ice at 0°C, heat it to boiling (100°C) and completely vaporize it?

(b) Which of the three processes above requires the most energy?

____

10. (a) Name five different forms of energy.

______________ ______________

______________ ______________

____

(b) Give examples of three different kinds of energy **changes** (other than those given in the lesson notes.)

____

____

____

11. 0.025 kg of water is in a copper container having a mass of 0.18 kg. If the water and container are cooled from 30°C to 10°C, how much heat is released by them?

12. A mercury thermometer is at 20.0° C and then is placed in hot water at 90.0° C. If the thermometer has 0.0240 kg of glass in it, and 0.00150 kg of mercury in it, how much heat would it absorb if it was warmed from 20.0° C to 90.0° C?

Answers to Self-Check Exercise #3, page 7.

1. $h = 25\ \text{kg} \times 4190\ \text{J/kg}\cdot{}^\circ\text{C} \times (40 - 10)^\circ\text{C} = 3.1 \times 10^6\ \text{J}$

2. $m = \dfrac{h}{c\Delta t} = \dfrac{5808\ \text{J}}{128\ \text{J/kg}\cdot{}^\circ\text{C} \times 302.5^\circ\text{C}} = 0.150\ \text{kg} = 150\ \text{g}$

End of Lesson H

PHYSICS UPGRADING
LESSON I

DO NOT send this lesson for correction

INTRODUCTION TO WAVES

Waves as Energy Carriers

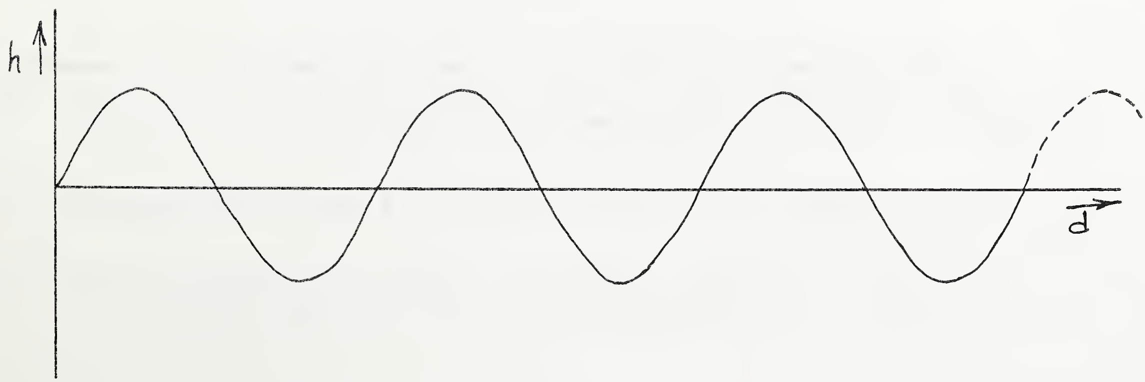
It is an absolutely lovely evening, warm and with not a whisper of a breeze. The sun's rays glow red against the clouds and through the distant trees. You are standing beside a small pond circled by overhanging trees. You start with a jerk as a sharp plop breaks the silence. A pulse of ripples spread out from where a squirrel dropped a small nut onto the mirror-like surface of the pond. A twig bobs up and down as the pulse passes it on its way to the shore.

Next day you are standing on the shore of a lake. Boats are idly resting against the piers. You hear in the distance the purr of a motor. Soon the boat is in sight with a single occupant. The prow points upward as the hull of the boat emits a long V of wave disturbances outward into the calm water. The boat passes the piers, boats motionless, the water smooth. Then suddenly the glassy surface bends upward as the wave arrives. The boats bob up and down tugging lightly at their moorings. The pulses slap against the shore one by one, then subside to a mere ripple.

You have just witnessed the formation and propagation of simple waves in a medium that we use and see everyday. You observed that the waves had a source, some input of energy — a falling object, a moving boat. You saw the wave travel at a certain definite speed. Neither wave pulse reached the shore instantly. You noticed its unique kind of disturbance as it passed objects. The objects bobbed up and down but did not move with the wave. You saw the wave expending energy as it caused movement of objects and created sounds on the shore.

Waves are disturbances in a medium, usually with a regular pattern, that carry energy through the medium. Such waves as those described above, where the medium moves at **right angles** to the wave's forward motion (medium moves up and down, wave moves forward), are called **transverse** waves. In a simple two-dimensional way we can represent a continuous train of waves as follows.

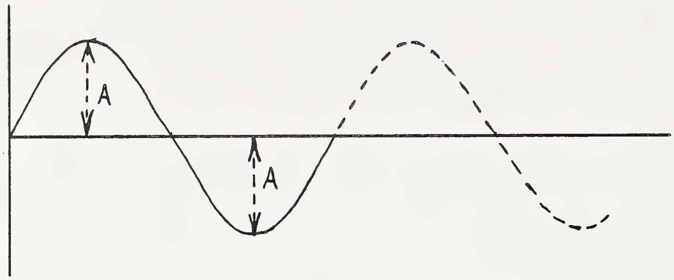
Fig. A



Characteristics of Waves

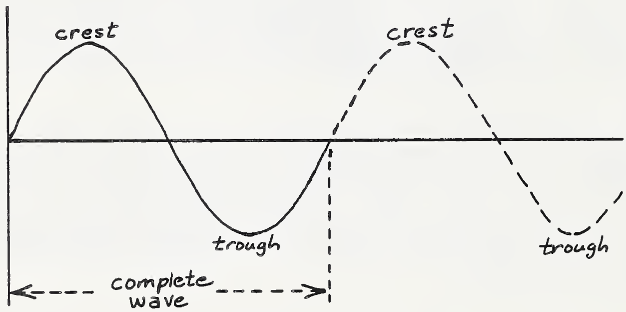
The wave has a number of features. It has an **amplitude** — the distance from the normally level surface of the medium to the peak or **crest** of the wave.

Fig. B



It has a **wavelength** (represented by λ - lambda, a letter of the Greek alphabet) — the distance between neighboring crests or troughs of a wave. A complete single wave consists of a crest and a trough (the lowest point in the disturbance).

Fig. C



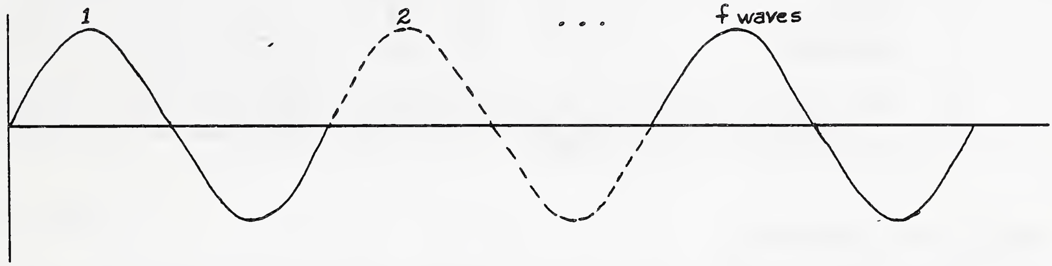
It has a **frequency** (in **Hertz** - **Hz** - the number of waves per second), the number of complete waves that are formed in a time of one second. The frequency of the wave is determined by the rate at which the source vibrates.

It has a **period** (T), the time it takes for a wave to be completed from beginning to end.

And finally a wave has a **speed** (**v**) which is the distance covered by a crest or trough (or any other part of a wave) in a unit of time.

There are some simple mathematical relationships that tie these characteristics of waves together. For example suppose the frequency of a wave is f and the wavelength is λ. This means that f waves are produced every second.

Fig. D



The wavelength of **each** of the waves is λ. If we multiply the number of waves by the length of each wave we obtain a value for the total distance of the wave train. The wave train has travelled for one second.

$$f \text{ waves/s} \times \lambda \text{ m} = f\lambda \text{ m/s}$$

Note that **m/s** are units of speed. (The term "waves" is a dimensionless or unitless quantity — it is simply a number obtained by counting). Thus $f\lambda$ is the speed of the wave and we can write:

$$v = f\lambda$$

This of course also means that

$$f = \frac{v}{\lambda} \quad \text{and} \quad \lambda = \frac{v}{f}$$

Another relationship is that between period and frequency. If the frequency of a wave is 10 Hz (10 waves per second) then **one** wave must be formed in 1/10 s. The period of that wave is 1/10 s. It takes 1/10 s to make one wave and 1 s to make 10 waves. Therefore we note that:

$$T = \frac{1}{f}$$

The units of f are **Hz** or **waves/s** or simply s^{-1} (which means 1/s) since "waves" has no units.

Thus $T = 1/f = 1/s^{-1} = s$ and the units of **T** are seconds.

To summarize:

Characteristic of wave	Symbol	Units
wavelength	λ	m
frequency	f	waves/s or Hz or s^{-1}
speed	v	m/s
period	T	s
amplitude	A	m

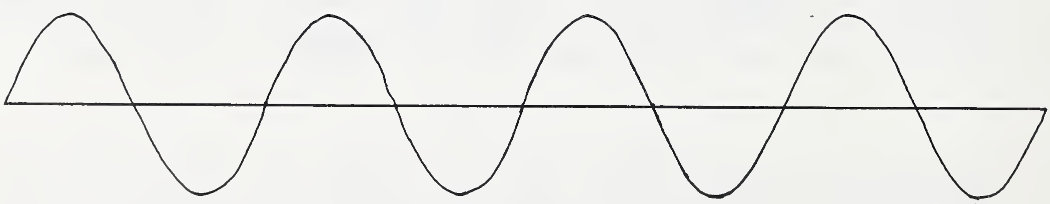
The last characteristic (A) is independent of the other four. It depends only on the **energy** produced by the source of the waves. It is the amplitude of the wave that carries the energy.

Self-Check Exercise #1

1. Match the following values with the wave characteristic it represents:

______	10/s	a.	wavelength
______	0.25 m	b.	speed
______	0.32 s	c.	frequency
______	10^{14} Hz	d.	amplitude
______	3.0×10^{8} m/s	e.	period
______	10^{-7} m		
______	300/s		

2. Using a cm ruler and calculations find the following values for the wave shown. The four wavelengths shown are formed in 0.25s.



(a) amplitude ______

(b) wavelength ______

(c) period

(d) frequency

(e) speed

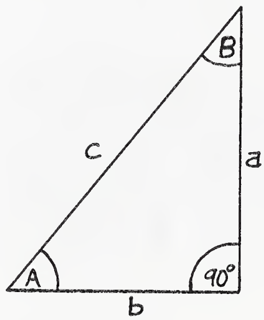
Answers to **Self-Check Exercise #1** on page 8.

Sine Waves

You were introduced to the trigonometric relationship called the **sine** of an angle in Lesson A. Recall that the sine of an angle is defined as

sin A $= \frac{a}{c}$. (See Fig. E)

Fig. E

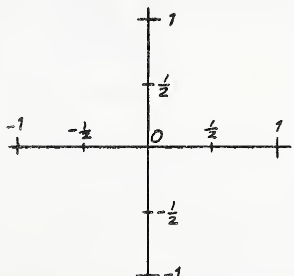


If **A** is 0° the side **a** in the figure is 0 and **sin A = 0.** If **A** is 90° the side **a** actually lies on the side **c** and **b** is 0. Thus

sin $90^\circ = \frac{a}{c} = \frac{c}{c} = 1$. Thus the sine of an angle varies from 0 to 1 while the angle varies from 0° to 90°.

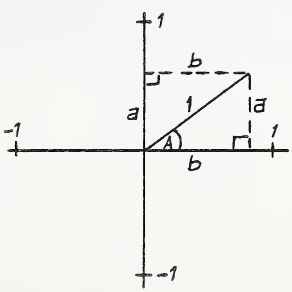
What happens if the angle becomes **larger** than 90°? Imagine a number line, or rather two number lines crossing each other at right angles. Numbers up and to the right are positive. Numbers down and to the left are negative.

Fig. F



On this set of number lines draw a line 1 unit long. This will form side **c** of a triangle. The sides **a** and **b** are formed if we draw perpendicular lines from the end of **c** to each of the number lines.

Fig. G

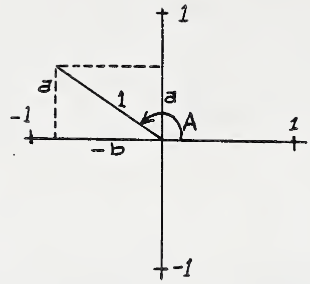


Now the sine of angle **A** is

sin A $= \frac{a}{1} = a$.

As **A** becomes larger **a** changes, becoming larger too. What happens when **A** gets larger than 90°? Look at the diagram. Again we can draw perpendiculars to the axes but note this time that the bottom side of the triangle is **–b** rather than **+b**. The sine of angle **A** is simply $\frac{a}{1} = a$.

Fig. H

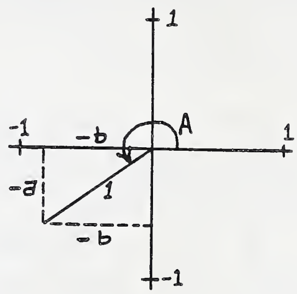


Note that as **A** goes from 0° to 90° the sine of **A** goes from 0 to 1. As **A** goes from 90° to 180° sin A goes from 1 to 0.

What happens if **A** is greater than 180°? Again refer to the diagram. Here both the triangle sides are negative. This means that

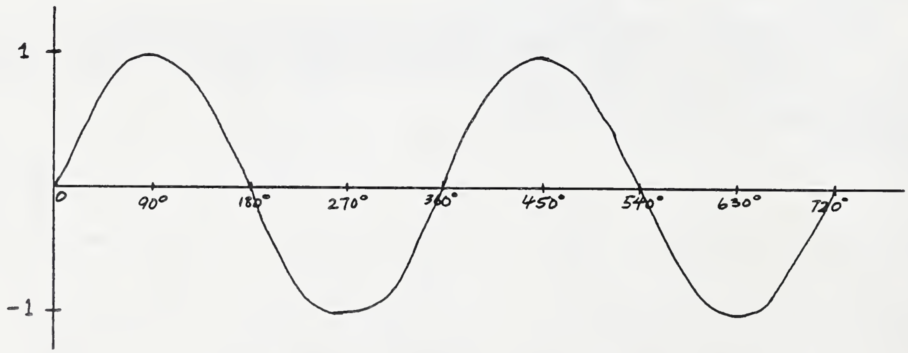
$$\sin A = \frac{-a}{1} = -a.$$

Fig. I



As **A** goes from 180° to 270° sin **A** goes from 0 to –1. We can now guess that as **A** goes from 270° to 360° (or 0° where we started from) **sin A** goes from –1 to 0. If we draw these values on a graph they would look like this:

Fig. J



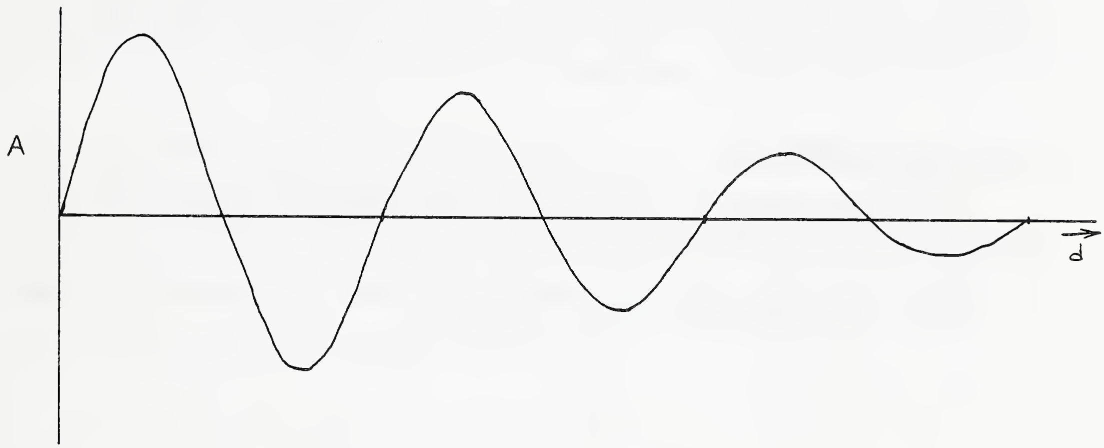
Where have you seen this shape before? It is exactly like the diagram for transverse waves on page 1! For that reason the transverse waves that we have been talking about are called **SINE WAVES** since their shape is like that of the graph of the sine function.

Wave Amplitude and Damping

We have already noted that the amplitude of a wave is the distance that the crest or trough of the wave is away from the undisturbed surface.

If you watch a water wave carefully as it travels across the surface of a lake you may notice that it becomes smaller and smaller (in amplitude, not wavelength) and gradually disappears. This gradual reduction in amplitude is called **damping**. Since the amplitude of a wave is related to energy, damping simply means that the wave is losing energy to the medium in which it travels. We can illustrate damping as follows:

Fig. K

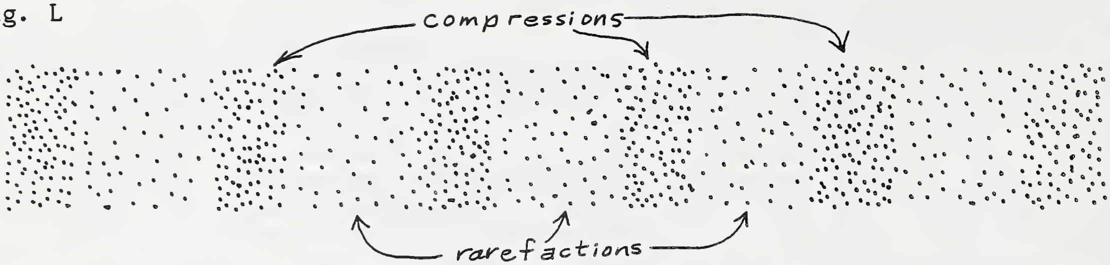


As the wave moves farther and farther from its origin its amplitude becomes less and less. Damping is usually caused by forces of friction or by the wave transferring its energy of motion to objects.

Longitudinal Waves

So far we have studied transverse sine waves, waves in which the motion of the particles or medium is at right angles to the direction of the waves' motion. There is another kind of wave where the vibration of the wave is in the same direction as the waves' motion. Instead of having high and low points in the wave we have, instead, **compressions** and **rarefactions**. At some places in the wave the particles of the medium are squeezed closer together than normal. This forms a compression. At other places the particles of the medium as they vibrate form a region where the particles are farther apart than normal. This is called a rarefaction. Compressions and rarefactions travel outward from a source just like crests and troughs do for a transverse wave. A longitudinal wave can be illustrated as follows in Fig. L.

Fig. L



One of the most familiar examples of longitudinal waves is sound waves in air. Sound waves are formed by some body vibrating back and forth. As it vibrates the molecules of the nearby air are alternately squeezed closer together and moved further apart. The resulting compressions and rarefactions travel outward from the source at about 344 m/s (when the air is at sea level and 20°C). This speed changes somewhat as the temperature and pressure of the air changes.

Self-Check Exercise #2

Assume that in all of the following situations the pressure is that at sea level and the temperature is 20°C.

1. What is the wavelength of sound waves from the **A** note of an organ pipe? (The frequency of **A** is 440 Hz).

2. What is the frequency of a sound wave that has a wavelength of 1.0 m?

Answers to Self-Check Exercise #2 on page 13.

Answers to Self-Check Exercise #1

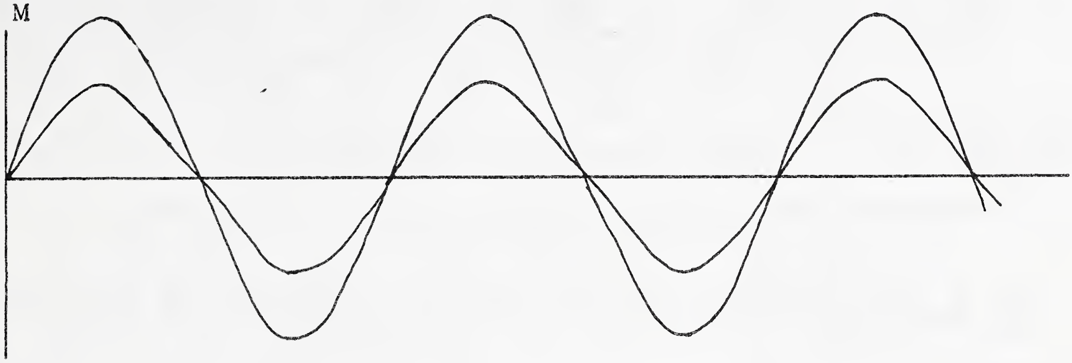
1. c, a or d, e, c, b, a or d, c

2. A = 1.4 cm, λ = 4 cm, T = 0.0625 s, f = 16 Hz, v = 64 cm/s.

Wave Phase and Interference

The term phase is used to describe how two (or more) waves are related to each other. For example, consider the following two waves:

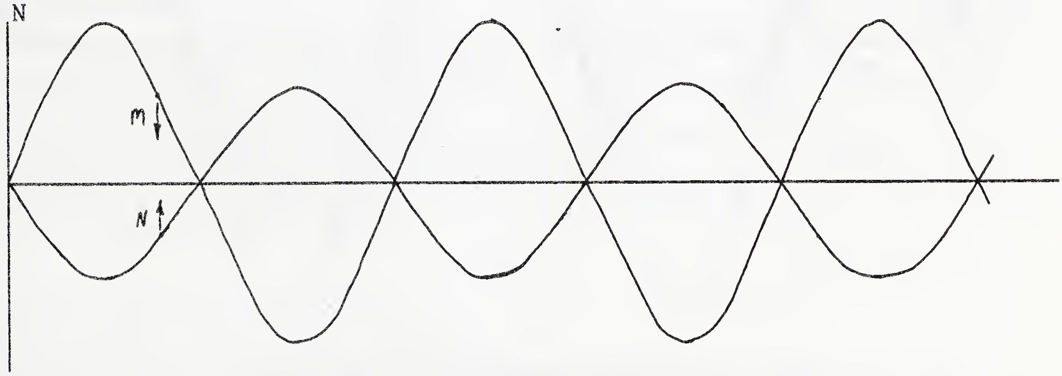
Fig. M



Note that when the higher amplitude wave is up the other wave amplitude is up too. When one wave displacement is 0° so is the other. Such waves are said to be **in phase.** There is a **phase difference** of 0°. Two waves are said to be in phase when the particles of the two waves at any point both have the same direction of displacement and the same direction of velocity.

Now consider the following two waves.

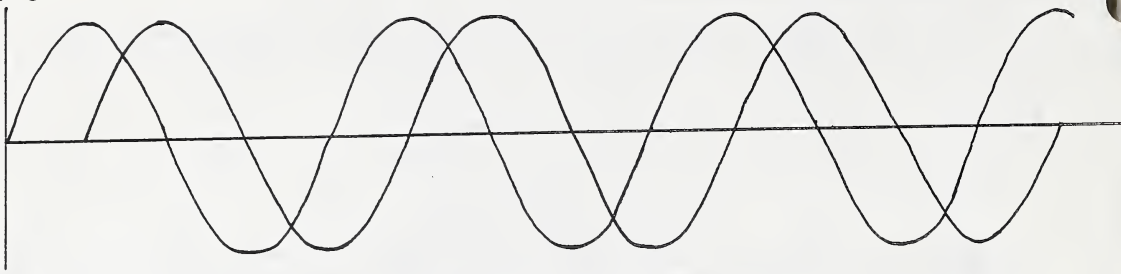
Fig. N



Note that when one wave particle is up the other is down. And note also that at a given time one particle **M** is moving downward while the other particle **N** is moving upward. Such waves are said to be **out of phase.** There is a **phase difference** of 180° between the two waves. Two waves are said to be out of phase when at a given point the particles in each of the waves have opposite displacements and are moving in opposite directions.

It is, of course, possible to have two waves that are out of phase by less than 180°. For example, see Fig. O.

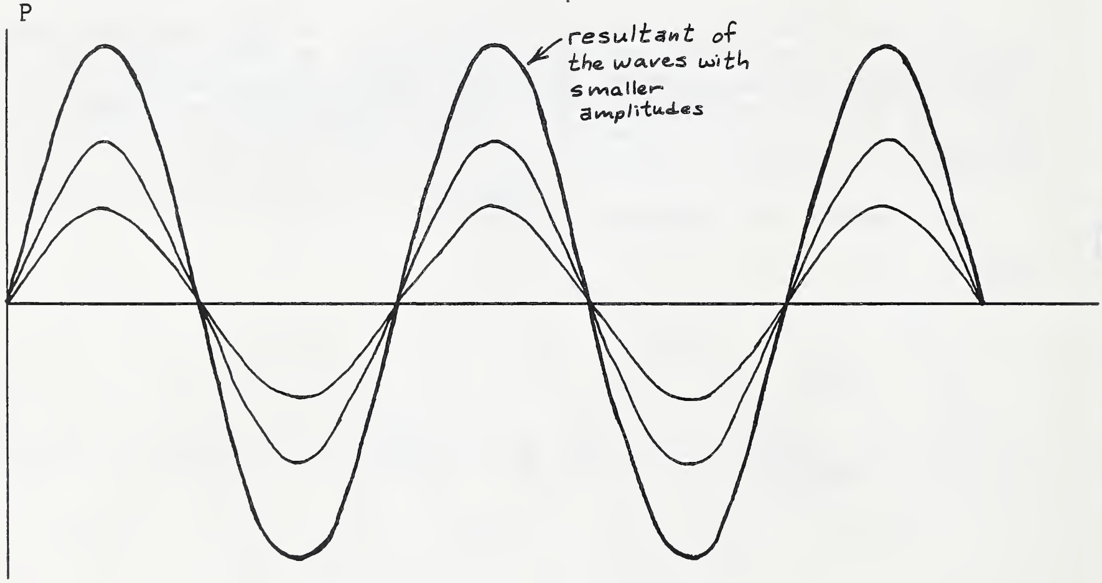
Fig. O



These two waves are out of phase by 90°.

It is interesting to observe what happens when we mix two waves that are in phase or out of phase. For example if we put the two waves shown in Fig. M together the amplitudes simply add together to make a larger wave.

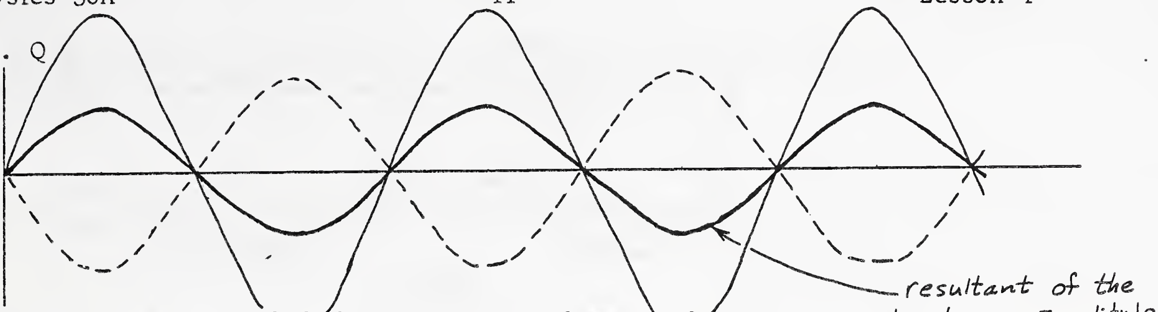
Fig. P



This is called **constructive interference** of waves.

However if we add the two waves of Fig. N together something different happens. The particles of the two waves are always moving in opposite directions. Therefore their motions will tend to cancel each other out. If the waves have the same original amplitude the two waves will completely disappear. If the amplitudes are different the result will be a wave with an amplitude very much reduced.

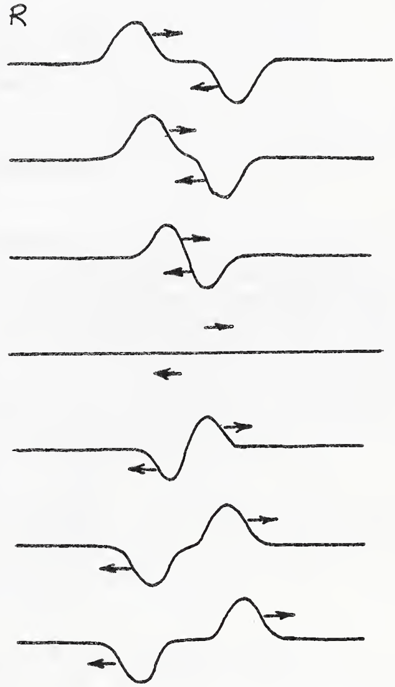
Fig. Q



This result is called **destructive interference** of waves.

It is interesting to note what happens when two wave pulses approach each other from opposite directions as shown in Fig. R.

Fig. R



Note that at the midpoint of their meeting they completely cancel each other and there is no deflection. Note also that the meeting of the two waves does not ultimately affect the shape of either one. They simply pass through each other causing some changes in deflection as they meet but then move on with the same shape as before.

Standing Waves

When a wave pulse on a line or a spring approaches a fixed attachment point it cannot, of course, continue moving. It will instead reflect from the surface but it will be upside down in comparison to its original shape. The pulse has undergone a 180° change of phase as it reflects.

This will produce a very interesting effect if a continuous train of waves of the right frequency are produced in a line whose end is fixed. A situation will be set up where the reflected waves are of the same wavelength as the original waves but they are moving in the opposite direction. Right at the point of reflection the waves will completely cancel but as the reflected wave travels along it will soon be in phase with the incoming wave and they will reinforce each other. The result will be a double amplitude. The situation is illustrated step by step on the following page.

Fig. S

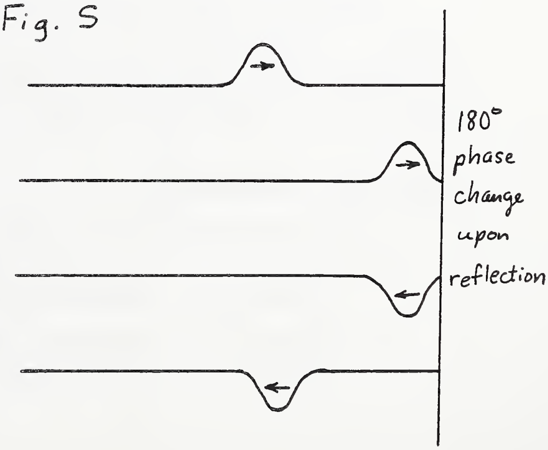
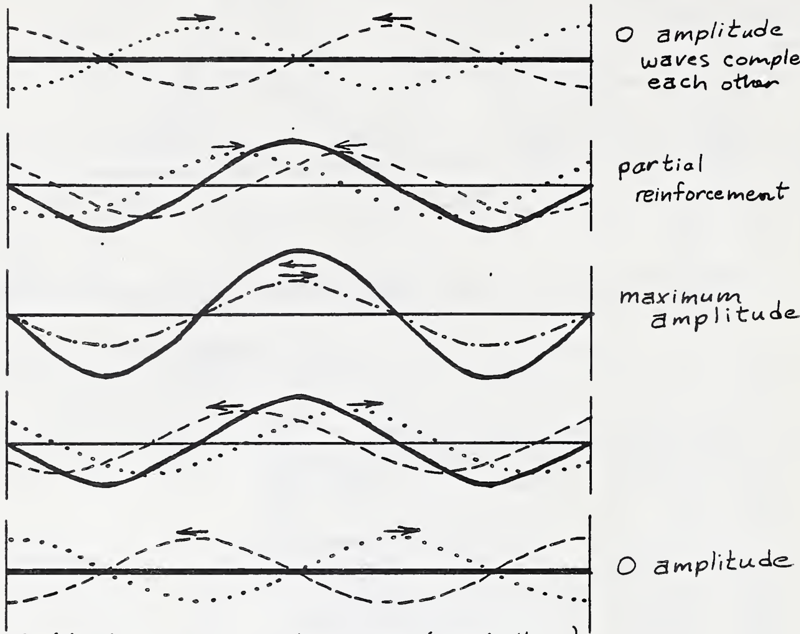


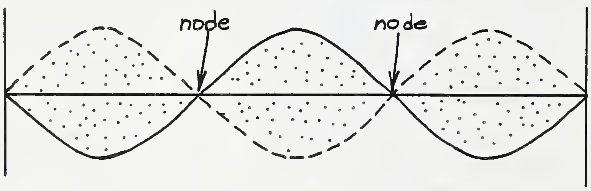
Fig. T



(Note how the waves travel past each other.)

The result is called a **standing wave** because it produces vibrations that appear to remain stationary with respect to horizontal movement. The material vibrates as follows:

Fig. U



The oscillations move back and forth between the lines shown. The points where there is no movement are called nodes.

Standing waves are important because they are responsible for the sound in organ pipes or for the sound from stretched strings in pianos, violins, guitars, etc. The vibrations in the strings are standing waves. Standing waves are also important in understanding the way electrons are bound to the atom as we shall see later in the Physics 30 course.

Self-Check Exercise #3

1. The amplitudes of two waves are 7 units and 3 units.
   (a) What is the maximum amplitude that can result from constructive interference? ________
   (b) What is the minimum amplitude that can result from destructive interference? ________

2. Draw a diagram showing a standing wave with three nodes (not counting the ends).

Answers to Self-Check Exercise #3 on page 24.

Answers to Self-Check Exercise #2, page 8

1. 0.782 m (about 78 cm) $\lambda = \frac{v}{f}$

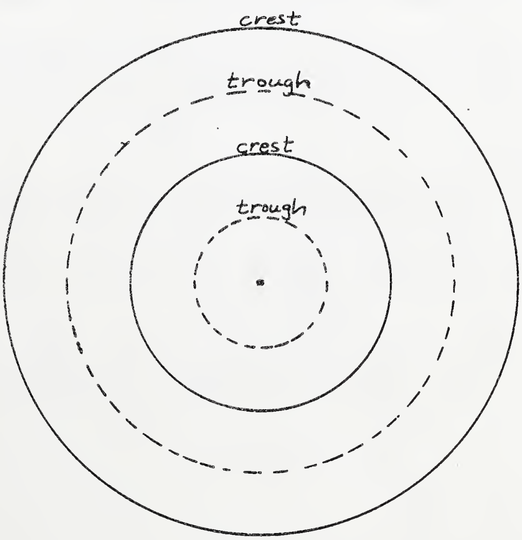
2. 344 Hz $f = \frac{v}{\lambda}$

Wave Fronts and Wave Rays

So far we have been using diagrams like Fig. A, page 1 to picture waves. However such a diagram has limitations. For example a diagram like Fig. A cannot show us how waves from a pebble dropped in water spread out in circles across the water surface. There is no way to show the circular pattern by using Fig. A. Similarly we cannot show how sound waves spread out in all directions from a source of sound. The problem is that the diagrams we have used are only in two dimensions (up and down, and across the page). But real waves are usually in **three** dimensions.

A way of solving the problem is to use the concept of **wave fronts.** A wave front is simply defined as the line joining points in a wave that are all in phase and whose motions all started at the same time. Usually we identify the crest or trough of a wave as a wave front. For example, the pebble dropping in the water would show wave fronts as follows:

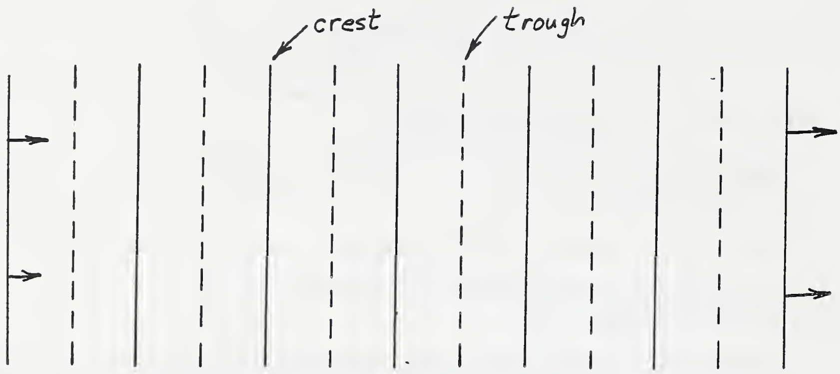
Fig. V



The diagram shows the situation somewhat as it would be seen from above. The dotted lines represent wave troughs and the solid lines represent wave crests. These wave fronts spread outward from the centre in larger and larger circles. At the centre when the water particles move upward a crest is formed. When the particles move downward a trough is formed. These crests and troughs move outward in continuously spreading circles.

Waves may also be formed with parallel fronts and they can be illustrated as follows:

Fig. W

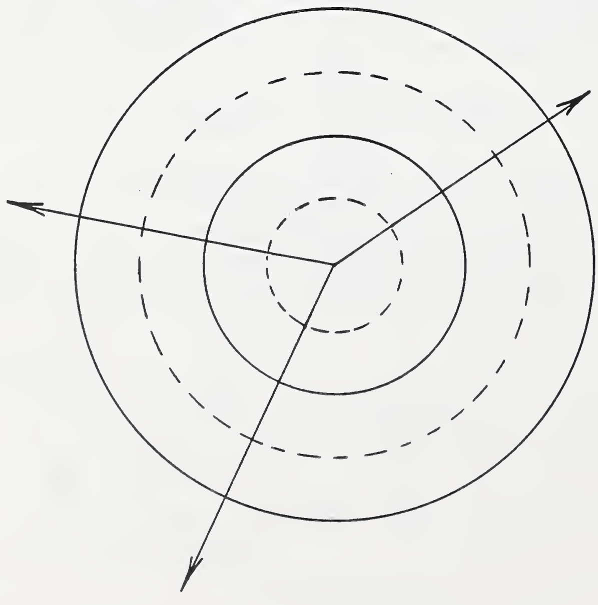


In this case we have a well-defined beam of waves.

There is yet another concept that is helpful to us in describing the behavior of waves. It is the concept of **wave rays**. A wave ray is simply a line drawn **perpendicular** to the wave fronts with an arrow showing the direction of the waves' motion. Most directly it gives us the direction of the waves' motion.

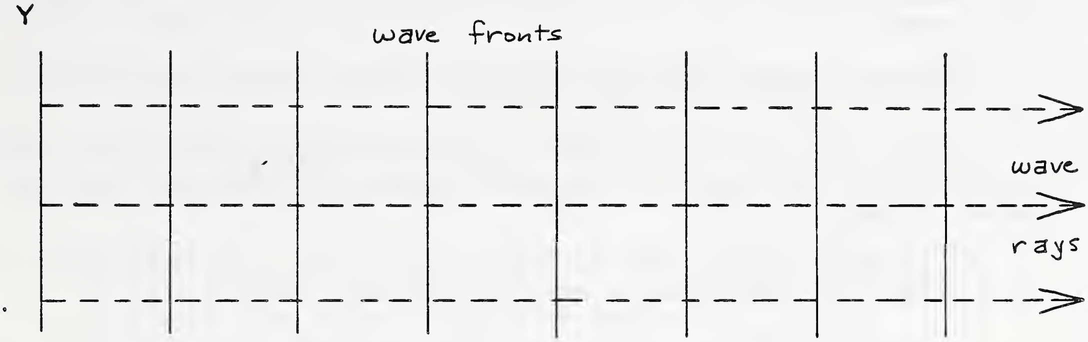
Wave rays for Fig. V could be drawn as follows.

Fig. X



Similarly wave rays could be drawn for Fig. W as follows:

Fig. Y



Waves as Energy Sources

One of the most common energy carriers are light waves or electromagnetic waves. We will have occasion to study these in Physics 30.

Another energy carrier is water waves. Water waves washing up against the shores of oceans tend to break down the rocks into sand and wear away the land. Energy is being used in this process. The energy in waves comes mostly from the wind that moves over the surface of the water. In recent years some thought has been given to harnessing the energy of water waves. A number of devices have been invented which will change the up and down wave motion to circular mechanical motion that can turn an electric generator. These devices include floats that move up and down with the waves or ships with hollow chambers in which the waves push and pull air through turbines. There are no doubt several effective ways in which wave energy will be tapped as a source of energy in the decades ahead.

Lesson Summary

Waves are regular energy-carrying disturbances in a medium.

Waves in which the motion of the medium is at right angles to the motion of the wave are called **transverse.** Waves in which the direction of motion of the medium is parallel to the direction of wave motion are called **longitudinal.**

All waves have a number of characteristics such as wavelength (λ), frequency (f), amplitude (A), period (T) and velocity (v).

The wave equation relates some of these characteristics:

$$v = f\lambda$$

Period and frequency are related by $f = \frac{1}{T}$.

The crest of a transverse wave is the highest displacement that the wave achieves above the undisturbed level and the trough is the lowest displacement.

Transverse waves are often called sine waves because their shape is similar to the graph of the sine function.

Damping of waves occurs when waves lose energy as they travel along in the medium. The result is a gradual reduction in amplitude until the waves disappear.

Longitudinal waves, such as sound waves, consist of compressions and rarefactions that move outward from the vibrating source.

Wave particles are in phase when the particles are moving in the same direction and have a similar displacement.

Wave particles are out of phase when the particles are moving in opposite directions and have opposite displacements.

Phase difference can vary anywhere from 0° to 360° out of phase. It simply means that one wave is a specified number of degrees behind another wave (one wavelength being 360°).

Constructive interference occurs when two waves in phase add together to increase wave amplitude.

Destructive interference occurs when two waves out of phase are added together to produce a lower amplitude than either.

Waves reflect from surfaces and in doing so change in phase by 180°.

Wave pulses passing through each other will change the amplitude at the meeting point but will pass on unchanged thereafter.

A standing wave is formed when waves of the same wavelength and frequency pass each other in opposite directions.

A wave front is a line joining points in a wave that are in the same phase and whose motion began at the same time. A wave ray is a line perpendicular to a set of wave fronts and indicates the direction of wave motion. Fronts and rays are helpful in accounting for the three dimensional characteristics of waves.

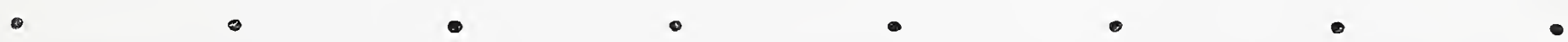
Since waves are carriers of energy it is possible to harness this energy for practical uses such as generation of electricity.

Exercises

1. What is a wave?

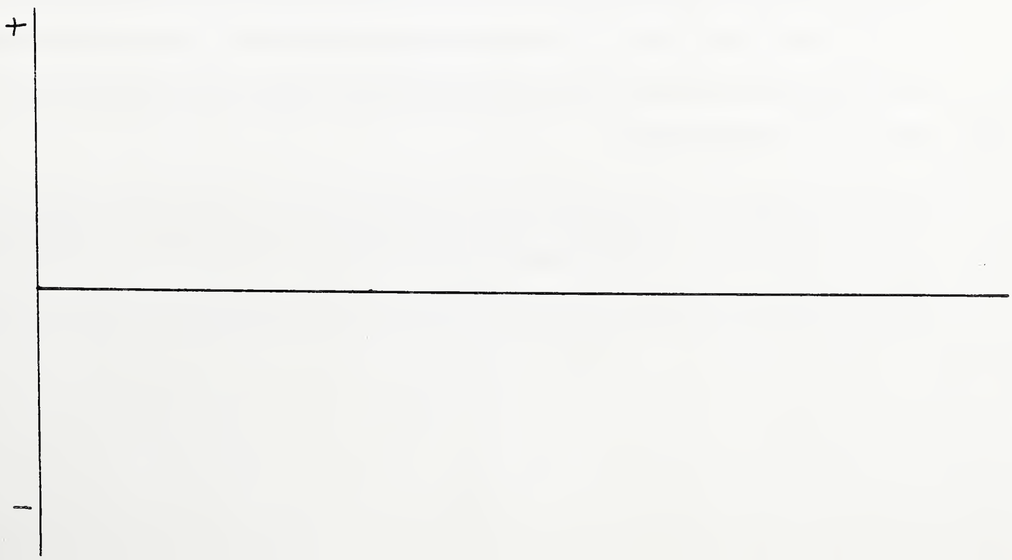
2. (a) In what direction do the particles of a medium vibrate when a transverse wave passes through the medium?

   (b) The following dots represent particles in a medium through which a transverse wave is passing. Attach an arrow to each dot to illustrate the direction in which each of the particles moves.



3. (a) What is meant by the term **amplitude?**

   (b) Sketch a positive pulse having an amplitude of 4.0 cm, followed by a negative pulse of an amplitude of 3.0 cm.



4. (a) What is **damping** as associated with waves?

(b) Draw a sketch which illustrates damping of a wave.

(c) What happens to the energy of a wave that is severely damped?

5. What is a longitudinal wave?

6. What is a compression?

7. In what ways are longitudinal waves similar to transverse waves?

____

____

____

8. Describe the following characteristics of waves:

(a) phase A way of describing how the motions of two waves are related to each other. When motions of particles are in the same direction at the same time and place, waves are in phase.

True or false? ________

(b) wavelength ________________________________

____

____

(c) period ___________________________________

____

____

(d) frequency ________________________________

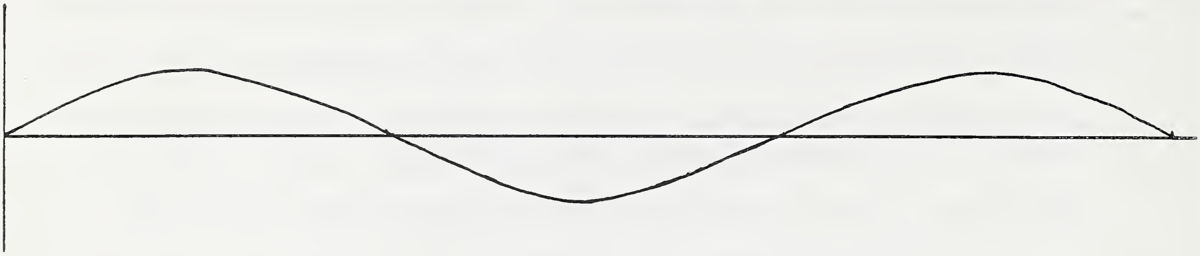
____

____

9. What is the frequency of vibration if the period is 0.4 s?

10. Calculate the period of vibration if the frequency is 60 Hz.

11. Using a cm ruler find or calculate the following characteristics of the wave shown. The time taken for one wavelength to form is 0.010 s.



(a) frequency

(b) wavelength ______________________

(c) amplitude ______________________

(d) period

(e) speed

12. As well as two waves being out of phase with each other, it is also possible for two points on a wave to be out of phase. Label two points, X and Y, that are 180° out of phase on the wave of #11.

13. In a particular medium, a source produces waves with a frequency of 150 Hz and a wavelength of 2.0×10^{-2} m. Calculate the speed of the waves through the medium.

14. The speed of sound in water is measured at 1.46×10^3 m/s. If a sound wave in water has a frequency of 8.0×10^2 Hz, what is the wavelength of the sound?

15. A wave travels through a medium with a speed of 25 m/s. If the wavelength is 1.25 m, what is the frequency of the wave?

16. (a) Which feature(s) of a wave are/is determined by the source (frequency, velocity, or period)?

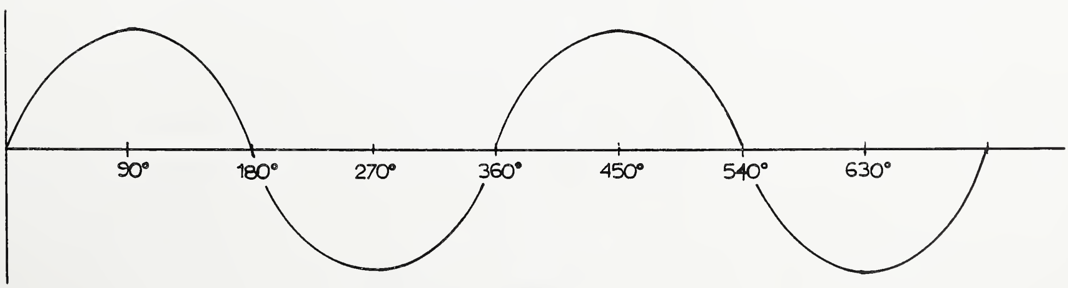
____

(b) Which features of a wave are determined by the properties of the medium through which it travels (frequency, velocity or period)?

(Hint: Make use of the relationship, $v = f\lambda$).

____

17. On the axes below draw a second wave that is 90° out-of-phase with the wave shown.



18. (a) In the space below draw a pattern for a standing wave that has only one node (not counting the end points).

(b) What is the wavelength of this standing wave?

____

(c) How many wavelengths would there be in a standing wave with **no** nodes between the end points? ____________________

19. What is a wave front?

____

____

____

20. What does a wave ray show?

____

21. A point source of vibrations vibrates at 1.5 Hz. The distance between crests is 4.5×10^{-2} m. Calculate the speed of the wave.

22. A source of waves is vibrating at a steady rate of 3 Hz. The waves formed spread out from the source, pass through oil and then into water where the waves travel faster than they do in oil. Explain what happens to each of the following wave characteristics and give reasons why it happens.

(a) Frequency ______________________________

____

____

(b) Wavelength ______________________________

____

____

(c) Period ______________________________

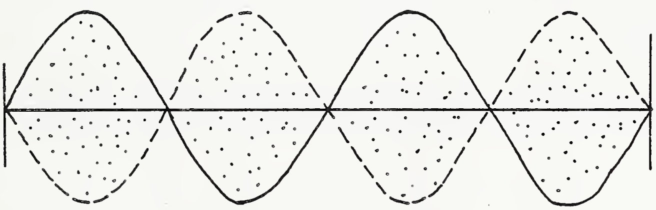
____

____

Answers to Self-Check Exercise #3

1. (a) 10 units (b) 4 units

2.



PHYSICS UPGRADING
LESSON J

DO NOT send this lesson for correction

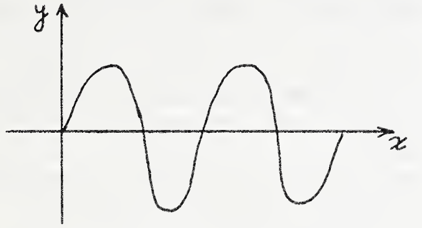
REFLECTION, REFRACTION, DIFFRACTION AND INTERFERENCE OF WAVES

Reflection

1. Wavefronts and Rays

A wave can be represented by a line showing a pattern in two dimensions, as illustrated in Fig. A. Such a representation of a wave is similar to wave motions exhibited by certain objects. For example, we could have a rope or cord form such a shape by rapidly moving one end of the rope up and down. Fig. A is a two dimensional representation of a wave because there are only two spatial dimensions needed to show the motion. These spatial dimensions are represented by the **x** and **y** axes.

Fig. A

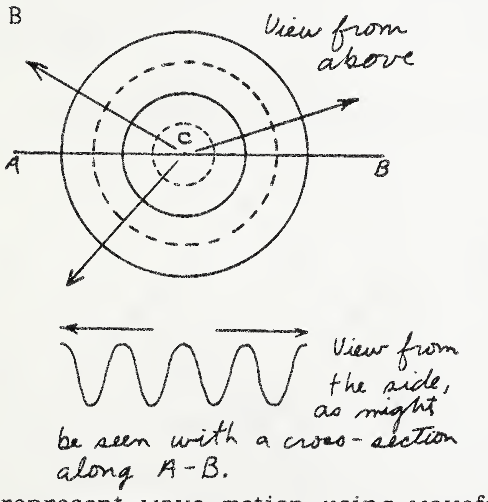


It often is easiest for us to show wave motion in two dimensions because "flat" surfaces such as pieces of paper usually are used for such illustrations. However, as you may be aware, most of the motions that we see and use occur in three spatial dimensions (which, for example, may be called the up-down dimension, the left-right dimension, and the forward-backward dimension). For example, water waves or ripples produced by an object falling in a pool appear to involve vertical (up-down) motions as well as motions outward from the source. This is illustrated by the two drawings in Fig. B. Note that the arrows show the directions of the wave motion. Because of the disturbance at point C, waves move outward from that point, as illustrated in the upper diagram, **(a)**. In part **(b)**, the wave motion in the vertical dimension is shown. Hence, by using two diagrams, it is possible to represent three dimensional wave motion.

Fig. B

(a)

(b)



The circular lines in part **(a)** of Fig. B join parts of the wave that were produced at the same time and that are in phase with each other. Such lines represent what are called **wavefronts.** The arrows showing the directions of motion of the wavefronts are known as **rays.** Rays are always perpendicular (at a 90° angle or a right angle) to wavefronts. In many of the diagrams in this lesson we will represent wave motion using wavefronts and rays. Although part of the description of the waves will not be included in those diagrams, this will not mean that it does not exist; it just allows us to study some important aspects of waves using simpler and relatively uncluttered diagrams.

Wavefronts may have various shapes, and sometimes they are quite complex. We will be concerned with wavefronts that have simple shapes. We have seen one shape in Fig. B: the circular wavefront. Such a wavefront can result from the activity of a point source. For example, the up-and-down motion of a relatively small object in a tank of water can result in waves that can be represented by circular wavefronts. Similarly, we can approximate the form of light waves from a small light bulb by using circular lines. These lines would represent in two dimensions spherical wavefronts produced by a point source of light. See Fig. C. On the line labelled 1 , a segment has been isolated and called **R–S.** Because the radius of circle 1 is relatively small, there is noticeable curvature in segment **R–S.** As the radius increases, the segments corresponding to **R–S** on the other wavefront lines show less and less curvature. When we reach 5 the segment **P–Q** shows very little curvature. At sufficient distance from source **B**, the small wavefront segments such as **P–Q** could be considered to be **plane** wavefronts; that is, they could be represented by a series of straight parallel lines, such as those shown in Fig. D. Plane wavefronts are those which can be considered to be "flat" (they have no curvature in them). In many situations, plane wavefronts can be assumed to exist at positions that are large distances from point sources. For example, because most stars are so far away, we can consider them to be point sources, and the light from them can be considered to be composed of plane wavefronts. In most of our discussions of reflection and refraction, we will be dealing with plane wavefronts.

Fig. C

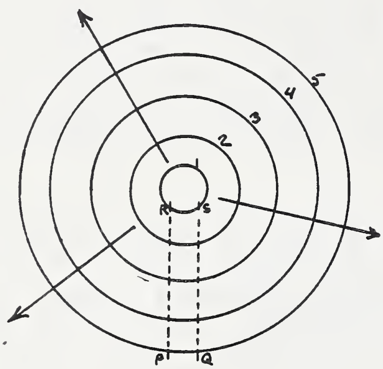
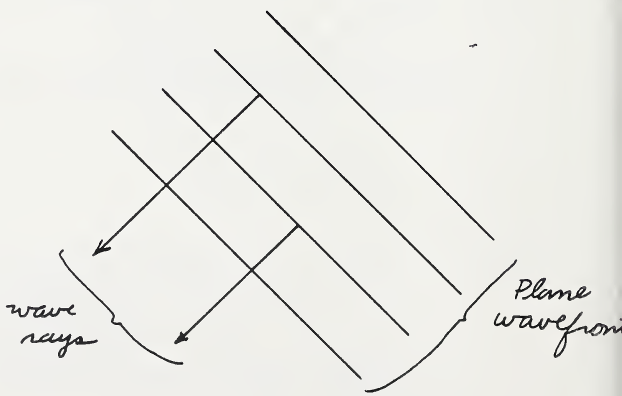


Fig. D



Self-Check Exercise #1

1. The directions of motion of wavefronts are shown by ________________ .

2. The angle between wavefronts and wave rays is always

________________________.

3. At large distances from point sources of waves, the curvature of the wavefronts is almost ______________________.

4. Wavefronts with no curvature are called ______________________________ wavefronts.

See page 4 for the answers.

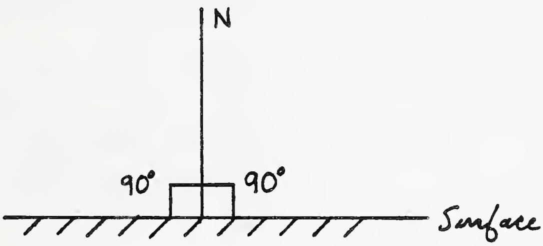
2. Normals and Tangents to Surfaces

Reflection of a wave occurs when a wavefront approaches and strikes a surface and then "bounces" away from or moves away from the surface. In order for this to occur, there must be some kind of interaction between the wave and the surface. We will be concerned with predicting the result of this interaction. Before we can do that, we will have to become familiar with some terms that will aid us in describing the motion of a wavefront with respect to a surface.

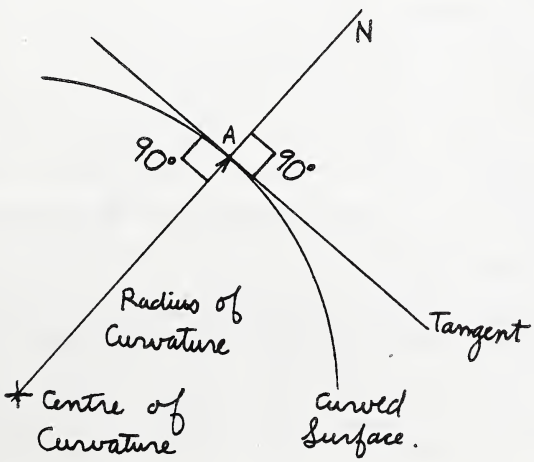
In part **(a)** of Fig. E, a flat or plane surface is shown. Perpendicular (or at a right angle) to that surface is a line labelled **N**. That line is called the **normal** to the surface.

Fig. E

(a)



(b)



The normal to any plane surface is always at 90° to the surface.

Many surfaces are not plane surfaces. They may involve some sort of curvature. Such a surface is illustrated in part (b) of Fig. E. The curvature of the surface is dependent on the length of the radius of curvature (the distance from the centre of curvature to the surface). The smaller the radius of curvature,the larger is the curvature. A line that is perpendicular to the radius of curvature line and just touches the curved surface at a pont **(A)** is known as the tangent to the surface at **A.** The line drawn perpendicular to the tangent (and labelled **N** in Fig. E) is the normal to the surface at **A.** Hence, we can find a normal at a point on a surface if the tangent at that point is known.

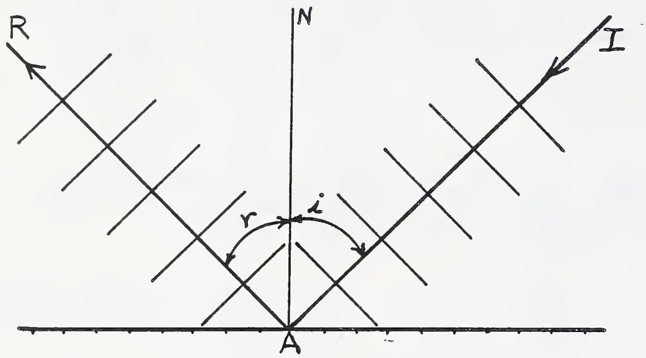
Answers to **Self-Check Exercise #1**, pages 2 and 3.

1. rays 2. 90° 3. zero 4. plane

3. Law of Reflection

The reflection of a plane wavefront from a plane surface is illustrated in Fig. F. The approaching or incident wavefronts are represented by the lines that are perpendicular to the line labelled **I**. Line **I** is a ray which is known as the incident ray for point **A** on the surface. When the incident wavefronts are reflected they move away from the surface in a direction indicated by **R**, the reflected ray. Note that the reflected wavefronts are perpendicular to the reflected ray. A normal, **N**, at point **A** is shown also. Two angles are shown between the two rays and the normal. Angle i is the angle of incidence. Angle **r** is the angle of reflection.

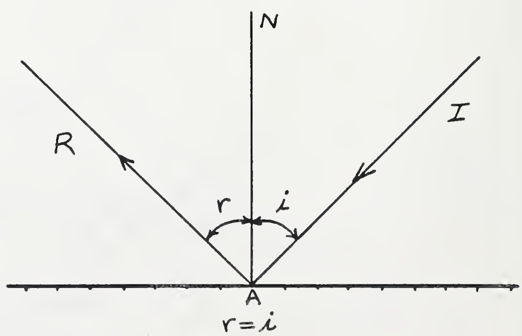
Fig. F



Usually when reflection is represented in diagrams, the lines for the wavefronts are omitted, and only the incident and reflected rays are used to represent them. Fig. G shows this.

The fundamental relationship for reflection is the following: with incident ray, reflected ray and normal all drawn in the same plane, the angle of incidence equals the angle of reflection. Referring to Fig. G, the law of reflection requires that angle i be equal to angle **r**: i = **r**. This law is true for plane or curved surfaces. Note that the angles are measured from the normal.

Fig. G



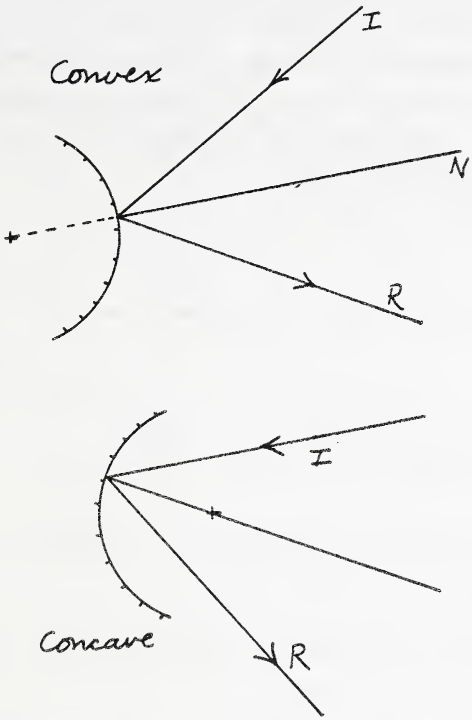
4. Reflection from Curved Surfaces

The law of reflection holds for any reflecting surface, but the results often are not as easily predicted when curved surfaces are involved as when plane surfaces are involved. The reason for this is that with a curved surface the direction of the normal (and consequently the angle of incidence) may not be immediately obvious. In Fig. H two circular reflecting surfaces are shown. In part **(a)** a convex surface is

Fig. H

(a)

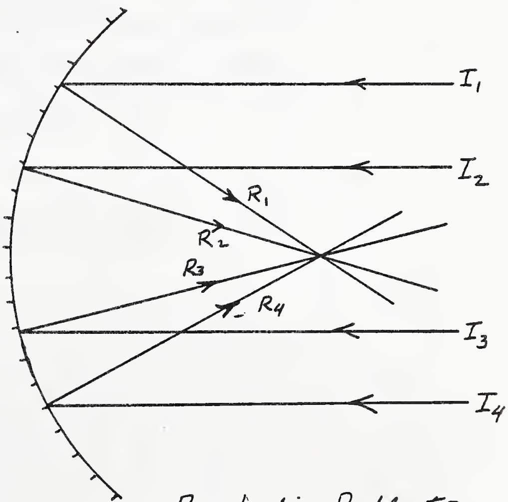
(b)



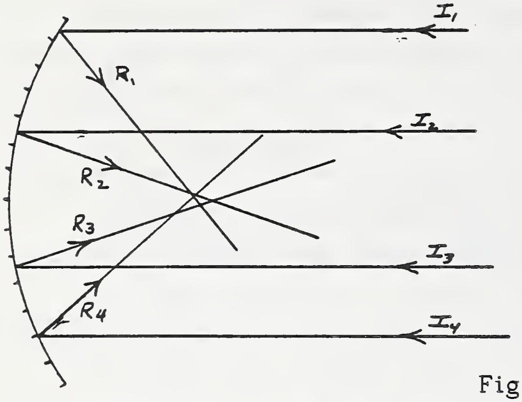
shown and in part **(b)** a concave surface is shown. (To remember which is which, note that the word concave includes the word "cave" and that the reflecting surface curves inward, like a cave.) For circular reflecting surfaces, it is not difficult to find the normal if you know where the centre of the circle is. The normal to a circular surface at a particular point lies along a line passing through that point and the centre of the circle. When the normal has been drawn, the incident ray and reflected ray can be drawn at the required angles, using the relationship **i = r.**

A special but important case of reflection from a curved surface is that of reflection from a parabolic surface. Parabolic surfaces may be used with such things as telescopic mirrors or the large "dish antennas" used in radio astronomy. The parabolic shape is useful because it will allow the reflection of plane wave fronts to a point. Fig. I illustrates this. The parallel incident rays become reflected rays that converge to a point. This may be useful because plane wavefronts from a relatively large area can be focussed on a smaller area with little distortion giving more intense images. Optical and radio astronomy telescopes use this principle.

Fig. I



Parabolic Reflector



A circular reflector does not produce focussing to a point. This is illustrated in Fig. J. Notice that the parallel incident rays do not result in reflected rays that all converge to one point.

Fig. J

Self-Check Exercise #2

1. The angle between the normal and tangent to a surface at a point always is ________________.

2. A ray showing the direction of approach of wavefronts to a surface is known as the ________________ ray.

3. A ray showing the direction of motion of wavefronts from a surface after reflection is known as the ________________ ray.

4. The angles of incidence and reflection are always measured between the incident and reflected rays and the ________________.

5. The law of reflection is that the angle of incidence always ________________ the angle of reflection.

6. Is the law of reflection true for all curved surfaces? ________________

7. A ________________ reflector reflects parallel incident rays to a point.

See page 8 for the answers.

NOW DO EXERCISE A ON PAGES 17 and 18

Refraction

The speed of a wave in a medium depends upon features of the medium. A wave may move faster in one medium than in another. When a wavefront moves from one medium to another, the change in speed that occurs results in a change in direction of the wavefront. This change in direction may be called refraction. Fig. K illustrates the refraction of a plane wavefront as it passes from one medium to another. The wavefront is represented by line W-F. In Medium 1, the speed of the wave is 1.5 cm/s. In Medium 2,

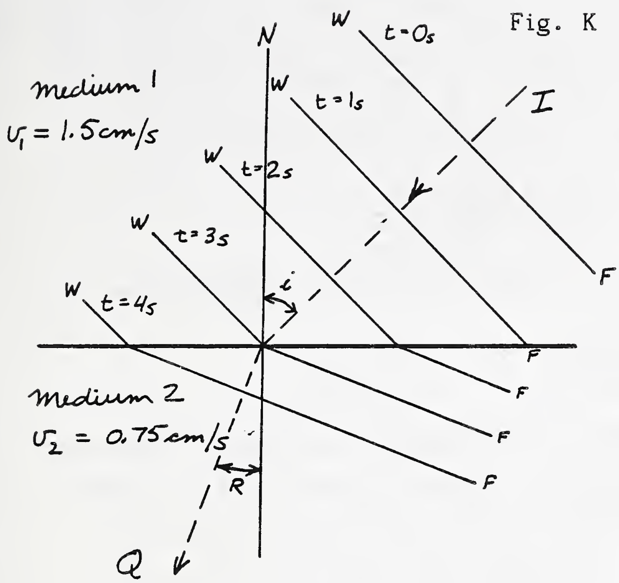


Fig. K

the speed of the wave is 0.75 cm/s. The positions of the wavefront are represented by the successive positions of line **W–F** at the times indicated as **t** = 0 s, **t** = 1 s, **t** = 2 s, **t** = 3 s. Note that for each 1 s interval, the wavefront moves 1.5 cm in Medium 1, and 0.75 cm in Medium 2. This results in a change in direction of the wavefront as it moves from Medium 1 to Medium 2. As indicated before, such a change in direction of a wavefront is known as refraction.

Often, to illustrate refraction more simply in diagrams, the lines representing the wavefronts are omitted, and just lines showing the boundary between the media, the incident ray **(I)**, the refracted ray **(Q)** and the normal **(N)** are shown.

As with reflection, we are not concerned with the details of how and why refraction occurs at this point. What is desired is some way of predicting the change in direction of a wavefront as it moves from one medium to another. In other words, we want a law of refraction, hopefully in a relatively simple and easy to use form. Such a law exists, and in the following notes we will show one way in which it can be derived.

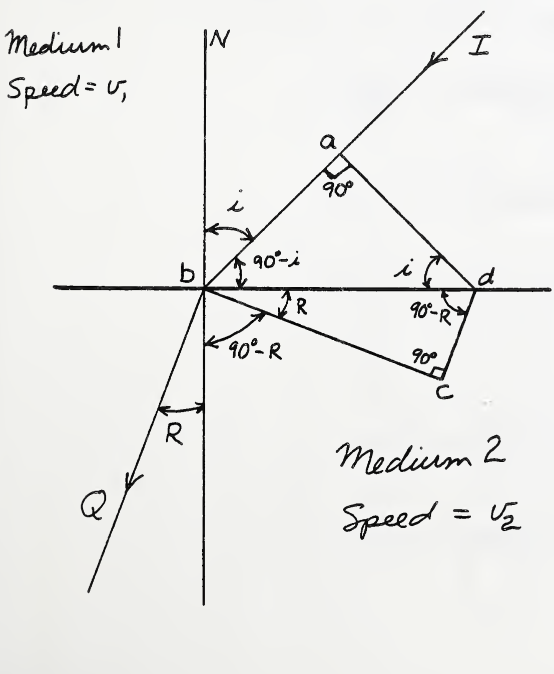


Fig. L

In Fig. L a wavefront in Medium 1 is represented by a line labelled **a–d**. In a period of time which we shall call t_0, the wavefront moves from the position of line **a–d** to the position of line **b–c**, in Medium 2. In other words, one end of the wavefront moves from **a** to **b** in a time t_0, and the other end moves from d to c in the same time t_0. Since the speeds in the two media are v_1 and v_2 for Medium 1 and Medium 2 respectively, we can express the distances **ab** and **cd** as shown on the following page.

$$ab = v_1 t_0$$

$$cd = v_2 t_0$$

For example, if $v_1 = 1.5$ cm/s, $v_2 = 0.75$ cm/s and $t_0 = 1.0$ s, then

$$ab = 1.5 \text{ cm/s} \times 1.0 \text{ s} = 1.5 \text{ cm}$$

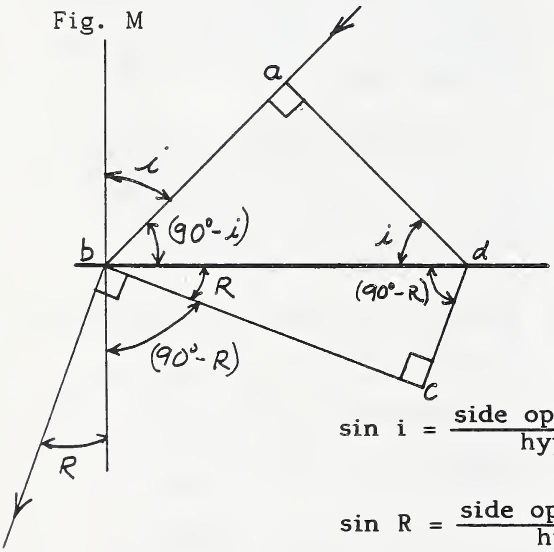
$$cd = 0.75 \text{ cm/s} \times 1.0 \text{ s} = 0.75 \text{ cm}$$

Answers to Self-Check Exercise #2, page 6

1. 90° 2. incident 3. reflected 4. normal 5. equals
6. yes

Now we have to use some trigonometry to obtain the law of refraction. Part of Fig. L has been redrawn in Fig. M. Two triangles with a common side are shown: triangle **abd** and triangle **bcd**. They are both right angle triangles, and both have the same hypotenuse: line **bd**. The sines of the angles i and R can be found in terms of the sides of the triangles by using the definition of the sine of an angle (the sine of an angle equals the ratio of the length of the side opposite the angle to the length of the hypotenuse of the triangle). If you are not clear about the meaning of the sine of an angle, you should see Lesson A.

Fig. M



$$\sin i = \frac{\text{side opposite angle i}}{\text{hypotenuse}} = \frac{ab}{bd}$$

$$\sin R = \frac{\text{side opposite angle R}}{\text{hypotenuse}} = \frac{cd}{bd}$$

From before, we know that $ab = v_1 t_0$ and $cd = v_2 t_0$

Forming the ratio of **sin i** to **sin R** we have

$$\frac{\sin i}{\sin R} = \frac{\frac{ab}{bd}}{\frac{cd}{bd}} = \frac{ab}{cd} = \frac{v_1 t_0}{v_2 t_0} = \frac{v_1}{v_2}$$

$$\boxed{\frac{\sin i}{\sin R} = \frac{v_1}{v_2}}$$

This is one form of the law of refraction. If we know v_1, v_2 and i, the value of the angle of refraction, R, can be found. Another form of the law involves what is known as the index of refraction. If the index of refraction for a wave moving from medium 1 to medium 2 is symbolized as n_{12}, then the law of refraction can be stated as

$$\boxed{\frac{\sin i}{\sin R} = \frac{v_1}{v_2} = n_{12}}$$

For example, if $v_1 = 1.5$ cm/s, and $v_2 = 0.75$ cm/s, then n_{12} would be found to be

$$n_{12} = \frac{v_1}{v_2} = \frac{1.5 \text{ cm/s}}{0.75 \text{ cm/s}} = 2.0$$

Notice that this is the index of refraction for the wave going from medium 1 to medium 2. If the wave were moving from medium 2 to medium 1, then the index of refraction n_{21} would be

$$n_{21} = \frac{v_2}{v_1} = \frac{1}{n_{12}}$$

Hence, the index of refraction for a wave moving from medium 1 to medium 2 is the inverse of the index for moving from medium 2 to medium 1.

To aid in solving problems involving refraction, it is useful to have a table giving values for the sines of angles. Such a table is shown below. Note that the values appear for angles from 0° to 90° in 5° intervals. More detailed listings of the values of sines of angles can be found in other trignometric tables. See page 19 of Lesson A.

Table 1

Angle (°)	Sine	Angle (°)	Sine
0	0	50	0.766 0
5	0.087 2	55	0.819 2
10	0.173 6	60	0.866 0
15	0.258 8	65	0.906 3
20	0.342 0	70	0.939 7
25	0.422 6	75	0.965 9
30	0.500 0	80	0.984 8
35	0.573 6	85	0.996 2
40	0.642 8	90	1.000 0
45	0.707 1		

Self-Check Exercise #3

1. The change in direction of a wavefront as it moves from one medium to another is known as ________________.

2. The change in direction of a wavefront as it moves from one medium to another occurs because the wavefront has different ____________________ in different media.

3. If the angle of incidence is 0°, then the angle of refraction must be ________.

4. In terms of the wave speeds in two media, the law of refraction may be given as $\frac{\sin i}{\sin R}$ = ______.

5. In terms of the index of refraction in going from medium 1 to medium 2, the index of fraction may be given as $\frac{\sin i}{\sin R}$ = ______.

6. The index of refraction in going from medium 2 to medium 1 is n_{21}. Give n_{21} in terms of n_{12}.

See page 12 for the answers.

The following examples illustrate the use of the law of refraction.

Example 1

A wave has a speed of 8.0 m/s in one medium and moves into another medium in which its speed is 24.0 m/s. If the angle of incidence is 15°, what will be the angle of refraction? (Use the values in Table 1, and estimate the angle of refraction to the nearest 5°.)

Given: $v_1 = 8.0$ m/s
$v_2 = 24.0$ m/s
$i = 15^\circ$

To find: R

Solution:

$$\frac{\sin i}{\sin R} = \frac{v_1}{v_2}$$

$$\sin R = \frac{v_2}{v_1}(\sin i) = \frac{24.0 \text{ m/s}}{8.0 \text{ m/s}}(\sin 15^\circ) = 3.0\ (0.2588)$$

$$= 0.776$$

From Table 1, the closest value to 0.776 would be the sine of 50° (0.7660).

$$\therefore R = 50^\circ$$

Example 2

When a wave moves from medium 1 to medium 2 the index of refraction is 2.5.

(a) What is the index of refraction in moving from medium 2 to medium 1?

(b) If the speed of the wave in medium 1 is 10.0 m/s, what is its speed in medium 2?

Given: $n_{12} = 2.5$
$v_1 = 10.0$ m/s

To find: (a) n_{21}
(b) v_2

Solution: (a) $n_{21} = \dfrac{1}{n_{12}}$

$$= \frac{1}{2.5}$$

$$n_{21} = 0.40$$

(b) $\dfrac{v_1}{v_2} = n_{12}$

$$v_2 = \frac{v_1}{n_{12}} = \frac{10.0 \text{ m/s}}{2.5} = 4.0 \text{ m/s}$$

OR

$$\frac{v_2}{v_1} = n_{21}$$

$$v_2 = v_1 n_{21} = 10.0 \text{ m/s } (0.40)$$

$$v_2 = 4.0 \text{ m/s}$$

DO EXERCISE B ON PAGES 19 and 20 NOW.

Answers to Self-Check Exercise #3, page 10.

1. refraction 2. speeds 3. 0° 4. $\frac{v_1}{v_2}$

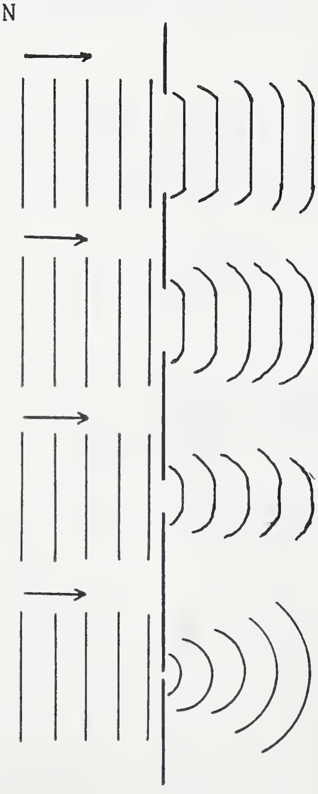
5. n_{12} 6. $\frac{1}{n_{12}}$

Diffraction

So far in this lesson we have been concerned with the change in direction of motion of wavefronts due to reflection and refraction. Wavefronts also can move around obstacles and change direction because of a third phenomenon known as **diffraction.** Diffraction may be described as the spreading of waves around a corner. Examples of diffraction of sound might be hearing the sound of a radio or TV set in another room through a doorway, or hearing the roar of a crowd near an open-air stadium. The extent to which spreading of waves occurs depends upon the wavelength of the waves and the size of the opening through which the waves pass. When the opening is much greater than the wavelength of the wave, diffraction effects are small. When the opening is near in size or smaller than the wavelength, diffraction effects are noticeable. Diffraction of sound through doorways or windows usually is significant because such openings have dimensions which are close in size to the wavelengths of sound waves.

Diffraction helps to explain why it is not possible to produce a very narrow wave "ray" by passing wavefronts through very narrow openings. As shown in Fig. N, as the opening gets smaller, the wavefronts bend more and more. Eventually, there is almost no evidence of plane wavefronts after they have passed through the opening. Hence, by making the opening smaller, we cannot produce a narrow beam or ray indefinitely.

Fig. N



This feature of wave behaviour can be understood in terms of Huygens' principle. Huygens' principle states that **any point on a wavefront can act as a point source for wavefronts produced in the direction of motion of the wave.**

Applying Huygens' principle to the situation of a wavefront meeting a narrow opening, we can see that the part of the wavefront that reaches the opening will act as a generator of wavefronts. If the opening is small in comparison to the wavelength, the part of the wavefront at the opening will act almost as a point source, producing almost spherical or circular wavefronts, as illustrated in the lower part of Fig. N.

Self-Check Exercise #4

1. The spreading of waves around a corner is known as ________________.

2. If an opening is small compared to the wavelength, then diffraction effects will be ________________.

3. The idea that any point on a wavefront can act as a point source of waves is known as ________________.

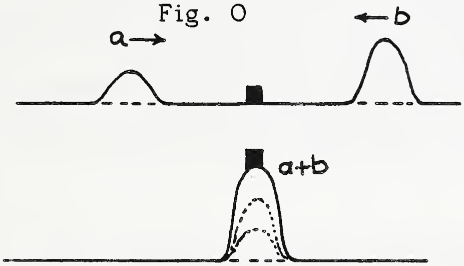
See page 16 for the answers.

Answers to Self-Check Exercise #5			
1. (a) 12.5 cm	(b) 3.5 cm	2. a node	3. an antinode

Superposition and Interference

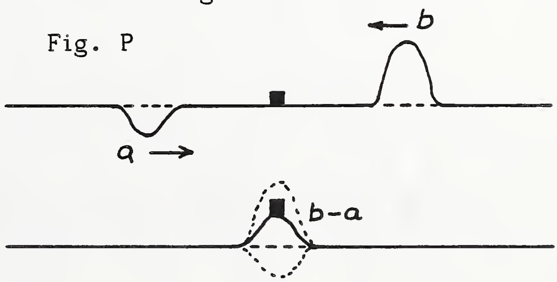
Superposition of waves refers to the idea that the effects of combining two waves of the same kind can be found by adding the effects of each wave individually. For example, suppose that we have a cork floating in water and a water wave causes it to cork up and down with an amplitude of 0.5 cm. If we were then able to apply another wave in phase with the first one with an amplitude of 1.0 cm, the cork then would have a maximum displacement of 0.5 cm + 1.0 cm = 1.5 cm. The amplitudes of the two waves could be added to obtain the net amplitude of the cork. Fig. O gives a simplified illustration of this idea. In the situation of Fig. O, the two pulses are "in phase" when they reach the cork because their peaks reach it at the same time.

Fig. O



If instead of the two peaks reaching the cork at the same time we have a peak and the lowest point of a trough meeting, then the resulting maximum displacement of the cork will be the difference between the two amplitudes. See Fig. P.

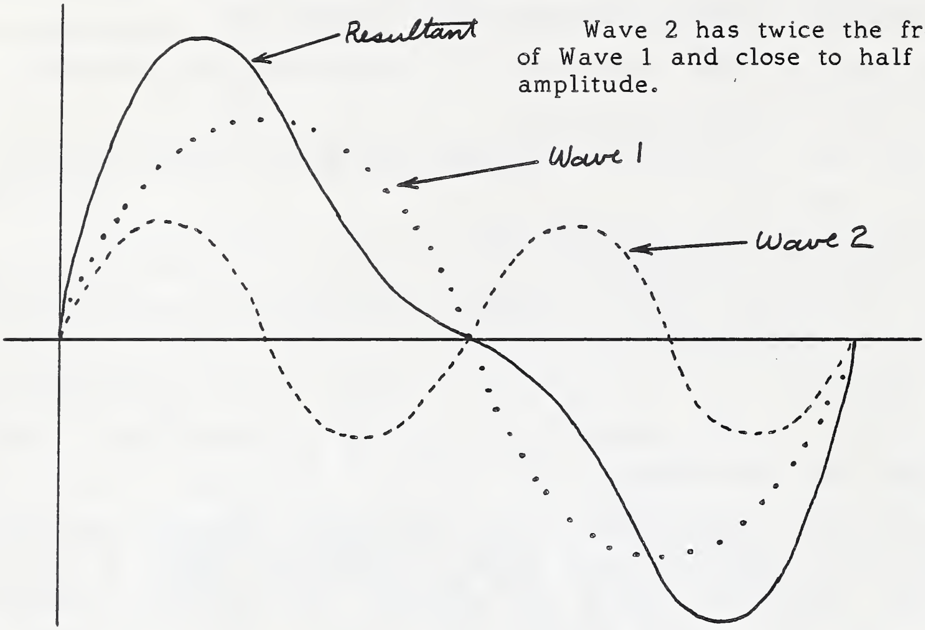
Fig. P



In this case we could say that the pulses were completely out of phase. When two waves of different frequencies and different amplitudes are combined, the principle of superposition can be used to find the resultant wave. This is illustrated in Fig. Q.

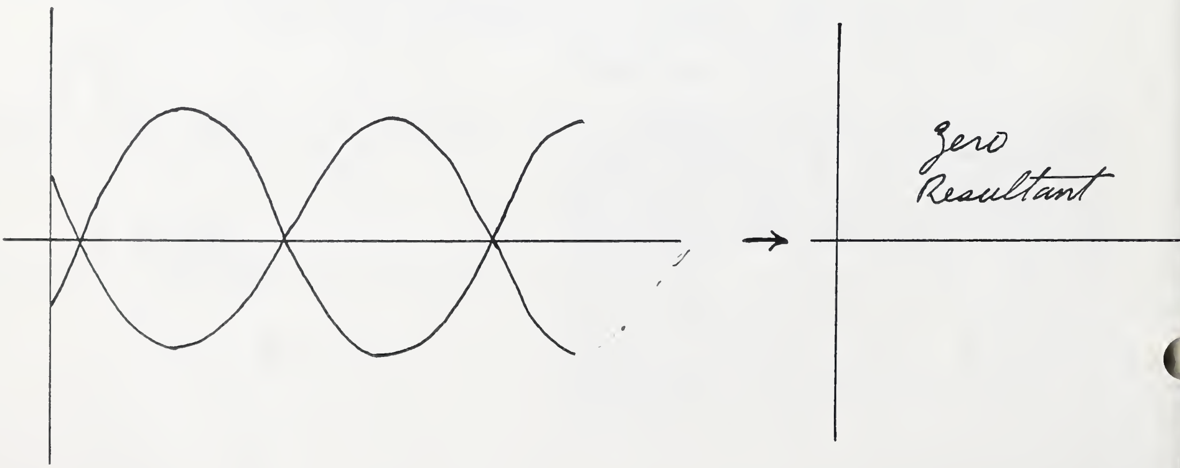
Fig. Q

Wave 2 has twice the frequency of Wave 1 and close to half the amplitude.



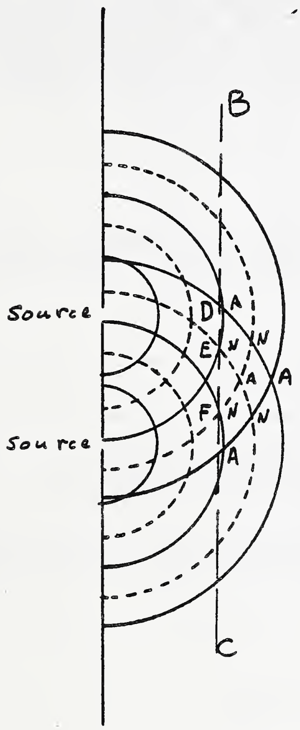
When two waves combine at a point to produce a resultant wave having an amplitude equal to the sum the amplitudes of the two waves at that point, then we can say that constructive interference has occurred. If two waves combine at a point to produce a resultant wave having an amplitude equal to the difference of the amplitudes of the two waves (and in the same direction as the wave of greater amplitude), then we can say that **destructive** interference has occurred. It is possible to have total destructive interference at some point. For example, if two waves combining at a point are completely out of phase and have equal amplitudes, their resultant at that point will be zero. See Fig. R.

Fig. R



When we apply the ideas of superposition, and constructive and destructive interference to wavefronts produced by vibrating sources, it is important to note that constructive and destructive interference may occur at various points, depending upon how the wavefronts interact. For example, in Fig. S the wavefronts produced by two vibrating sources are illustrated. The solid lines represent the positions of the peaks, and the dashed lines represent the positions of troughs. Each point where two solid lines meet, or where two dashed lines meet is a point at which maximum constructive interference occurs. Each point at which a dashed line meets a solid line is a point at which total destructive interference occurs. Points at which total destructive interference occur are sometimes called **nodes**. Some nodes in Fig. S have been labelled with N's. A node may be defined as a point having a zero displacement continuously.

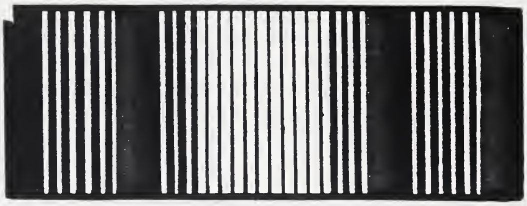
Fig. S



A point going up and down with maximum amplitude is called an **antinode**. Antinode positions correspond to points in Fig. S at which two solid lines meet or two dashed lines meet. Some of these points have been labelled with A's in Fig. S.

When wavefronts from two vibrating sources interfere, the resulting pattern is called an interference pattern. Interference patterns may be similar to the pattern shown in Fig. S, or they may have different forms. They are very useful and important in the study of physics.

For example, suppose we shine a beam of light on two slits close together (corresponding to the two points called "source" in Fig. S). An interference pattern from the light waves will form on the other side of the slits. Now place a plane surface in the interference pattern to act as a screen. It is represented by the line BC shown in Fig. S. The point D is very close to an antinode - a point of maximum **constructive** interference. We see a **bright** area on the screen. At points E and F near a node where maximum **destructive** interference occurs **dark** areas are formed. If we extended the screen and included more waves, the screen would show a series of light and dark bands gradually fading out towards its ends. If we looked at the screen from a view point near the slits we would see an interference pattern as follows:



This pattern can be used to get information about the light waves. If we know the distance between the slits and the distance of the screen from the slits we can, for example, calculate the wavelength of the light waves that are used to form the pattern.

Self-Check Exercise #5

1. Two waves having amplitudes of 4.5 cm and 8.0 cm are combined. What is the amplitude of the resultant for (a) total constructive interference and (b) total destructive interference?

   (a) ______________________________

   (b) ______________________________

2. A point at which total destructive interference occurs is called

   ________________.

3. A point at which there is a maximum value of displacement continuously is called ________________.

   See page 13 for the answers.

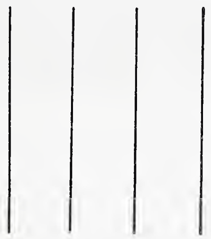
Answers to Self-Check Exercise #4		
1. diffraction	2. great	3. Huygens' Principle

DO EXERCISE C ON PAGES 21 and 22 NOW.

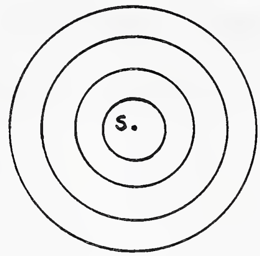
Exercise A - Reflection

1. Draw wave rays in the following diagrams illustrating wavefronts.

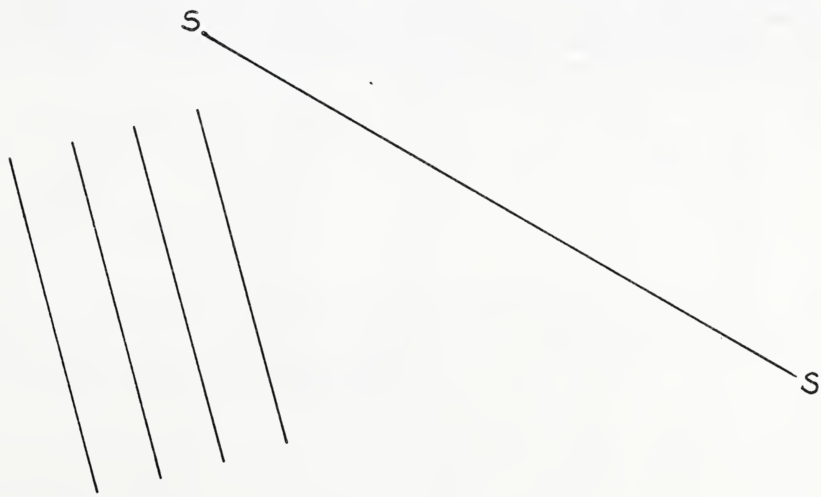
   (a) Plane wavefronts moving to the right.

   (b) Circular wavefronts moving outward from point S.

   (c) Plane wavefronts moving toward surface S–S.

2. A wavefront strikes a plane surface and is reflected from it. The angle between the incident ray and the **surface** is 18°.

   (a) What is the angle of incidence?

   ____

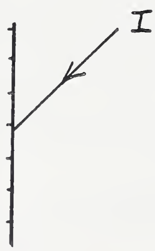
   (b) What is the angle of reflection?

   ____

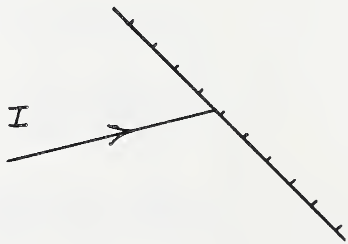
   (c) Draw a diagram for the situation described above showing the reflection, including incident and reflected rays and the normal, but omitting lines representing wavefronts. The line representing the surface has been drawn for you.

3. Draw reflected rays for each of the incident rays (represented by I) shown in the following diagrams. Label the reflected rays with R. For the curved surfaces, draw the tangent and normal lines also, and label them with **T** and **N** respectively.

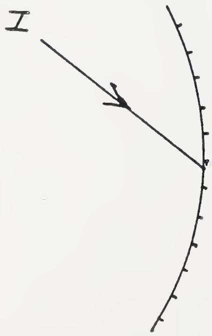
(a)



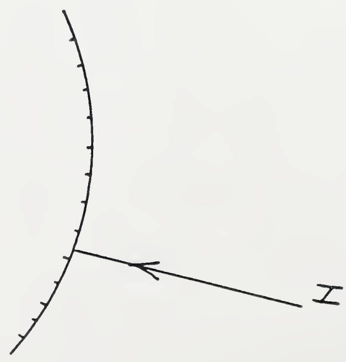
(b)



(c)



(d)



Exercise B - Refraction

1. A wave moves from one medium in which its speed is 30 cm/s to another in which the wave speed is unknown.

   (a) If the index of refraction in going from the first to the second medium is 2.5, find the wave speed in the second medium.

   (b) Calculate the index of refraction in moving from the second to the first medium.

2. The angle of incidence of a wavefront on a boundary between two media is 65°. If $n_{12} = 2.91$, find the angle of refraction.

3. The index of refraction for light moving from air to water is 1.33.

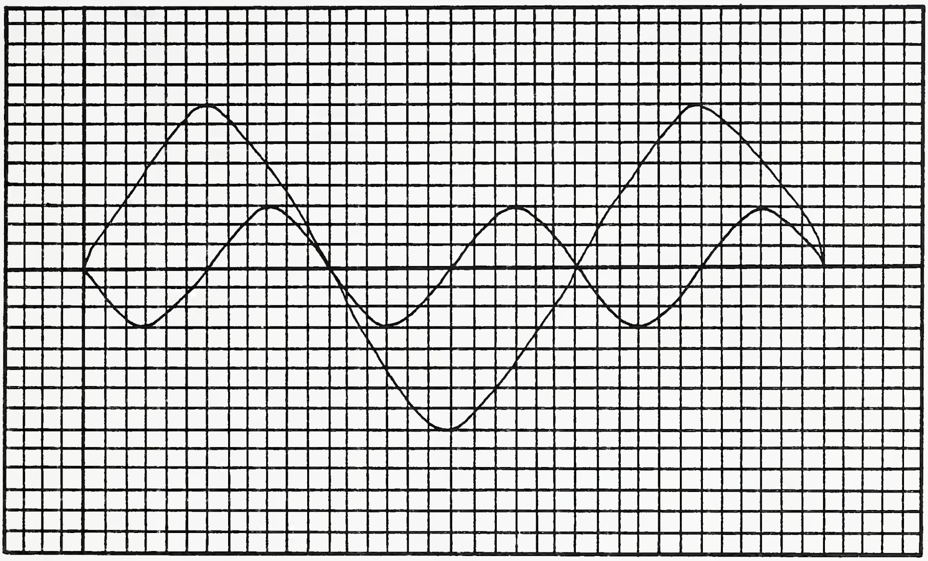
   (a) If the angle of incidence of light on a water surface is 70°, what will be the angle of refraction?

(b) If the speed of light in air is 3.0×10^8 m/s, what is the speed of light in water?

(c) Draw a diagram showing the incident ray, refracted ray and normal for the situation described in 3(a).

Exercise C - Diffraction and Interference

1. Two waves are illustrated below. Draw a diagram (on the same set of axes) showing their resultant by superposition.



2. Diffraction effects are easily noticeable with sound because the wavelengths involved are close in size to many of the openings involved in our every day experiences. Why is diffraction of light waves (wavelength range: about 4.0×10^{-7} m to 7.0×10^{-7} m) not so easily noticeable? Why can we not "see around corners" as easily as we can "hear around corners"?

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3. Two point sources of water waves produce circular wavefronts which interfere. Describe the motion of a small cork placed at (a) a node and (b) an antinode.

(a) node - _______________

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(b) antinode - ______________________________

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End of Lesson J